

Similarity & Congruence (H)

A collection of 9-1 Maths GCSE Sample and Specimen questions from AQA, OCR, Pearson-Edexcel and WJEC Eduqas.

Name:	SOLUTIONS
Total Marks:	

1. Triangle ABC is isosceles with $AB = AC$.

The line BP bisects $\hat{A}BC$.

The line CQ bisects $\hat{A}CB$.

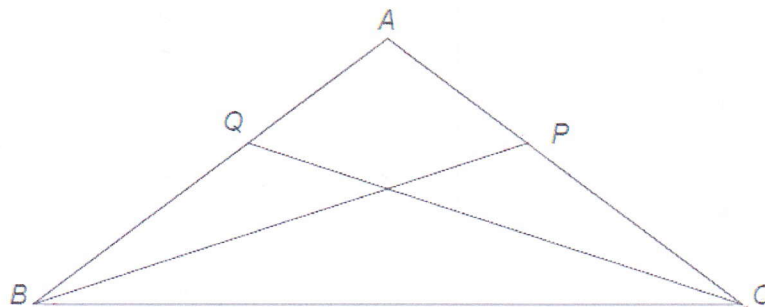


Diagram not drawn to scale

Prove that triangle BCP and triangle CBQ are congruent.

You must give reasons to support your statements.

$\angle PCB = \angle QBC$ (base angles of isosceles triangle equal)

so $\angle PBC = \angle QCB$ (angles were bisected)

$BC = BC$ so BCP and BCQ congruent (ASA).

[5]

2. Steph is solving a problem.

Cube A has a surface area of 150 cm^2

Cube B has sides half the length of cube A

What is the volume of cube B?

To solve this problem, Steph decides to

- 3 ☒ halve the surface area
- 2 ☒ calculate the square root of the answer
- 1 ☒ then divide by 6
- 4 ☒ then cube this answer to work out the volume.

$$150 \div 6 = 25$$

$$\sqrt{25} = 5 = \text{side length A}$$

$$5 \div 2 = 2.5 \text{ (B)}$$

$$2.5^3 = \text{Vol}$$

Evaluate Steph's method.

correct steps but wrong order (ordered correctly above)

[2]

3. Which of these is not used to prove that triangles are congruent?

Circle your answer.

SSS

SAS

AAA

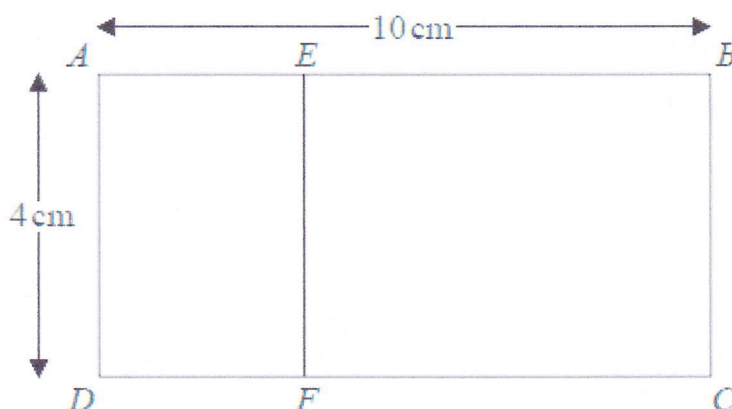
RHS

[1]

4. Rectangle $ABCD$ is mathematically similar to rectangle $DAEF$.

$AB = 10$ cm.

$AD = 4$ cm.



Work out the area of rectangle $DAEF$.

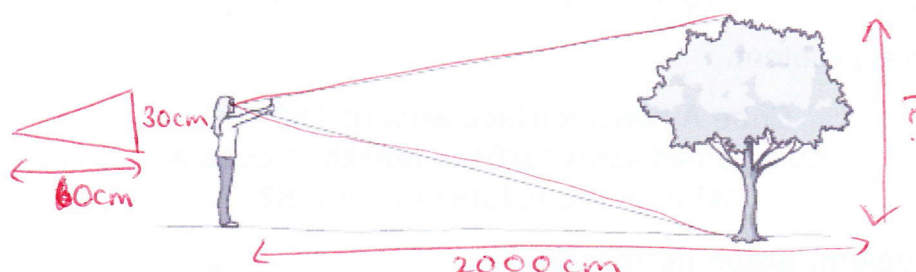
$$\text{Area } ABCD = 40 \text{ cm}^2$$

$$40 \div 6.25 = 6.4 \text{ cm}^2$$

$$\begin{aligned} \text{SF (length)} &= \frac{10}{4} = 2.5 \\ \text{so SF (area)} &= 2.5^2 = 6.25 \end{aligned}$$

$$\dots\dots\dots 6.4 \dots \text{ cm}^2 [3]$$

5. (a) Anna estimates the height of a tree.



Anna holds a ruler vertically so the height of the tree is exactly covered by the ruler.

She is 20 metres from the tree.

The ruler is 30 cm long.

The horizontal distance from her eyes to the ruler is 60 cm.

UNITS!!

Calculate an estimate of the height of the tree.

$$SF = \frac{2000}{60} = 33.\dot{3}$$

$$30 \times 33.\dot{3} = 1000 \text{ cm}$$

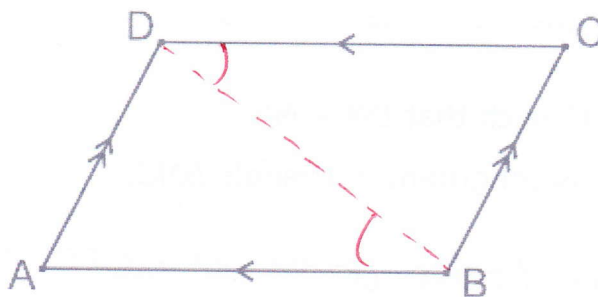
(a) 10 m [3]

(b) Give two reasons why this method may not be suitable to estimate the height of a very tall building.

- Likely to be inaccurate because of very large scale factors + distances.
- would have to stand far away from building.

[2]

6. ABCD is a parallelogram.



Prove that triangle ABD is congruent to triangle CDB.

$$DC = AB$$

$$\angle CDB = \angle DBA \text{ (alternate)}$$

$$DB = DB$$

SAS $\Rightarrow$ congruent.

[3]

7. Solid A and solid B are mathematically similar.

The ratio of the surface area of solid A to the surface area of solid B is 4:9

The volume of solid B is 405 cm^3 .

Show that the volume of solid A is 120 cm^3 .

	A : B
area	4 : 9
length ($\sqrt{}$)	2 : 3
Volume (cube)	8 : 27

$$405 \div 27 = 15$$

$$8 \times 15 = 120 \text{ cm}^3$$

[3]

8. Two spheres have radii in the ratio 5 : 3

Circle the ratio of their volumes.

5 : 3

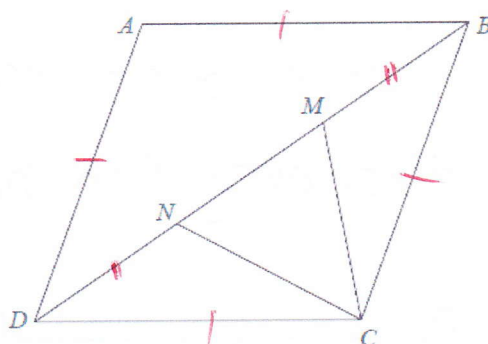
15 : 9

25 : 9

125 : 27

[1]

9. $ABCD$ is a rhombus.



M and N are points on BD such that $DN = MB$.

Prove that triangle DNC is congruent to triangle BMC .

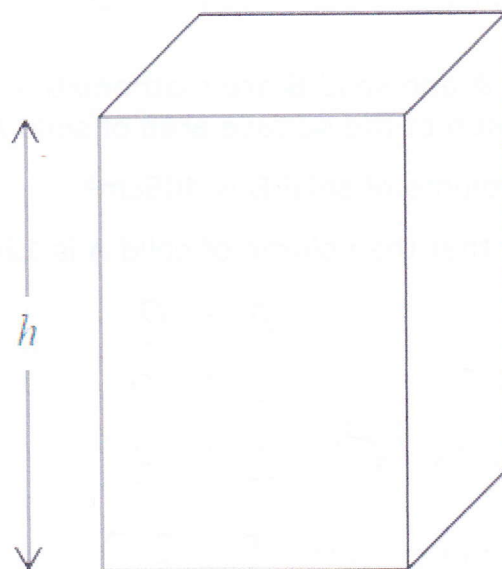
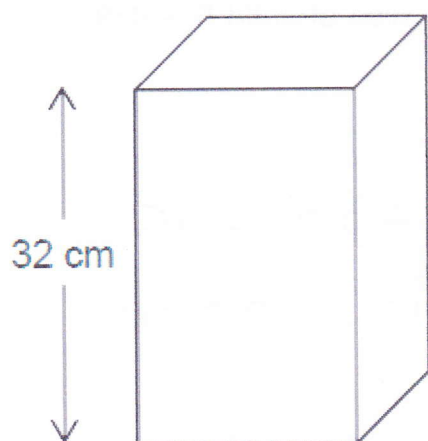
$DN = MB$
 $\angle CDN = \angle CBM$ (base angles of isosceles $\triangle$ equal)
 $DC = BC$ (rhombus)
 $\therefore \triangle DNC = \triangle BMC$ (SAS)

[3]

10. Two boxes are made with card.

The boxes are similar cuboids.

The smaller box has height 32 cm



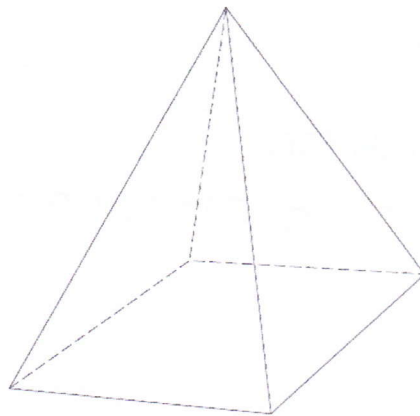
It takes 44% more area

Work out the height, h , of the larger box.

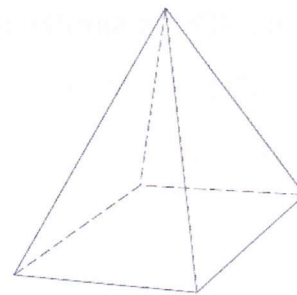
$$\begin{array}{l} \text{small : large} \\ \text{Area} \quad 1 : 1.44 \\ \text{length}(\sqrt{}) \quad 1 : 1.2 \\ 32 \times 1.2 = \underline{38.4 \text{ cm}} \end{array}$$

[4]

11. Two similar pyramids A and B have surface areas 180 cm^2 and 80 cm^2 respectively.



Pyramid A



Pyramid B

$$180 \div 80 = 2.25$$

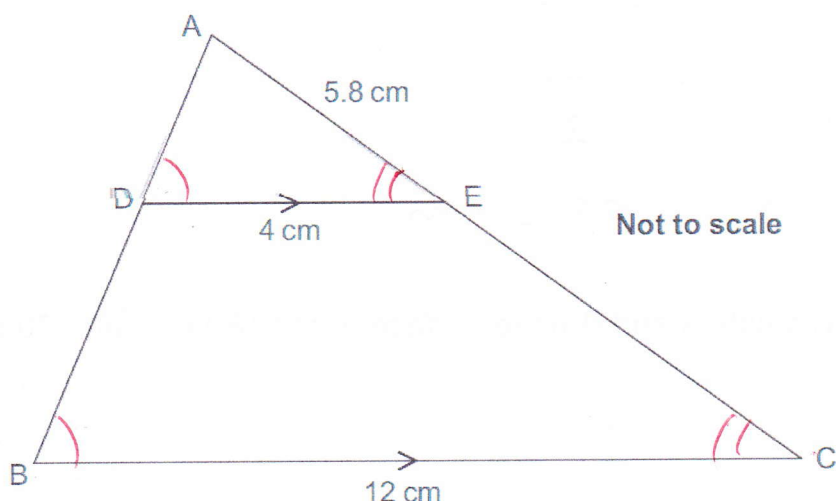
The volume of pyramid A is 810 cm^3 .

Show that the volume of pyramid B is 240 cm^3 .

$$\begin{array}{l} \text{Area} \quad 2.25 : 1 \\ \text{length} \quad 1.5 : 1 \\ \text{volume} \quad 3.375 : 1 \end{array} \quad \begin{array}{l} A : B \\ 810 \div 3.375 = \underline{240 \text{ cm}^3} \end{array}$$

[5]

12. In the diagram BC is parallel to DE.



a) Prove that triangle ABC is similar to triangle ADE.

$$\begin{aligned} \angle ADE &= \angle ABC \text{ (corresponding angles equal)} \\ \angle AED &= \angle ACB \quad " \quad " \quad " \\ \angle A &= \angle A \end{aligned}$$

AAA = similar

[3]

b) Calculate the length of AC.

$$SF = \frac{12}{4} = 3 \quad 5.8 \times 3 = 17.4 \text{ cm}$$

b) 17.4 cm

[2]

c) Find the ratio

area of quadrilateral DBCE : area of triangle ABC.

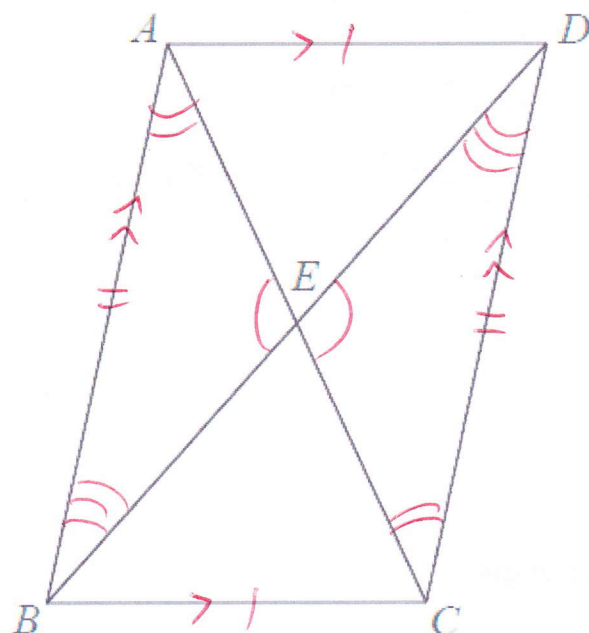
$$SF \text{ length} = 3 \text{ so } SF \text{ area} = 9$$

$$\text{area ADE} : \text{area ABC} = 1 : 9$$

$$\text{area DBCE} : \text{area ABC} = 8 : 9$$

c) 8 : 9 [3]

13 $ABCD$ is a parallelogram.



E is the point where the diagonals AC and BD meet.

Prove that triangle ABE is congruent to triangle CDE .

$\angle AEB = \angle CED$ (vertically opposite)
 $\angle EAB = \angle EDC$ (alternate) $\rightarrow DC = AB$ (parallelogram)
 $\angle ABE = \angle EDC$ (") $\rightarrow AA = E$ (congruence)

ASA = Congruent [3]

14 Mark has made a clay model.

He will now make a clay statue that is mathematically similar to the clay model.

The model has a base area of 6cm^2

The statue will have a base area of 253.5cm^2

Mark used 2kg of clay to make the model.

Clay is sold in 10kg bags.

Mark has to buy all the clay he needs to make the statue.

How many bags of clay will Mark need to buy?

MODEL : STATUE

6 : 253.5

$$\sqrt{253.5 \div 6} = 6.5$$

$$(2 \times 6.5^3) \div 10 = 54.925$$

$\uparrow$ SF length

so 55 bags needed...

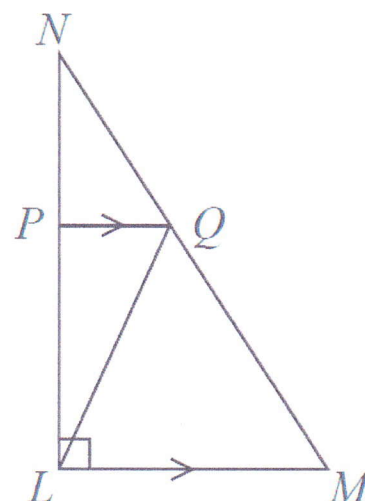
[3]

15.

$$16 \div 8 = 2 \quad PL = 2 \times NP$$

$$\text{Area } LMN = 8 \times (2+1)^2 = 72 \text{ cm}^2$$

$$\text{so Area } LQM = 72 - 16 - 8 = \underline{48 \text{ cm}^2}$$



LMN is a right-angled triangle.

Angle $NLM = 90^\circ$

PQ is parallel to LM .

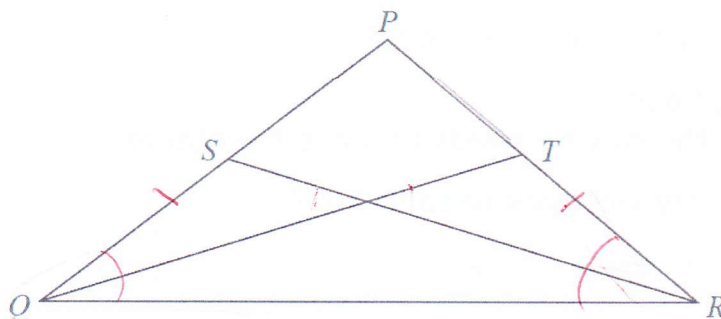
The area of triangle PNQ is 8 cm^2

The area of triangle LPQ is 16 cm^2

Work out the area of triangle LQM .

..... 48 cm^2 [4]

16.



$PQ = PR$.

S is the midpoint of PQ .

T is the midpoint of PR .

Prove triangle QTR is congruent to triangle RSQ . $QR = QR$

$\angle PQR = \angle PRQ$ (isosceles)

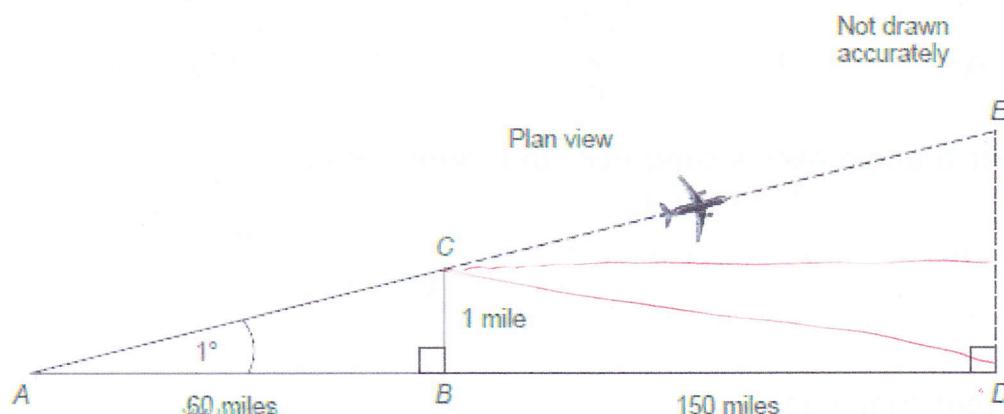
$TR = SQ$ (bisects PQ/PR)

$\therefore$ congruent (SAS)

[3]

17. The pilot of an aircraft wants to fly from A to D.

The aircraft flies from A to E, 1° off course.



a) The distance BC is 1 mile.

$$210 \div 60 = 3.5$$

Work out the distance DE.

..... 3.5 miles [2]

b) How should the aircraft have turned at C to fly directly towards D?

Tick a box.

1° clockwise

☐

between 1° and 2° clockwise

☐

2° clockwise

☒

more than 2° clockwise

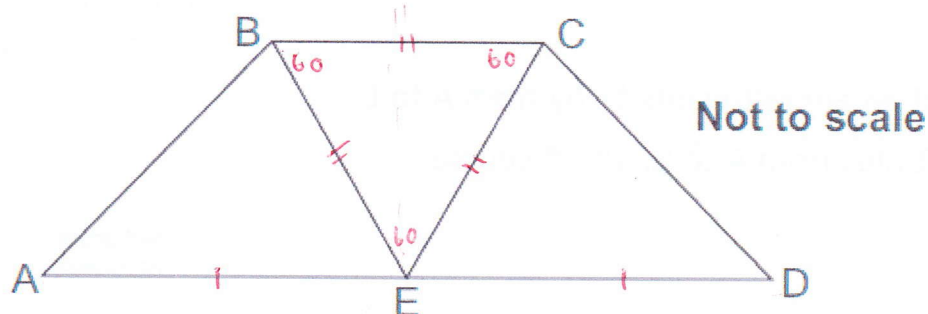
☐

[1]

18. The diagram shows trapezium ABCD.

E is the midpoint of AD.

BCE is an equilateral triangle.



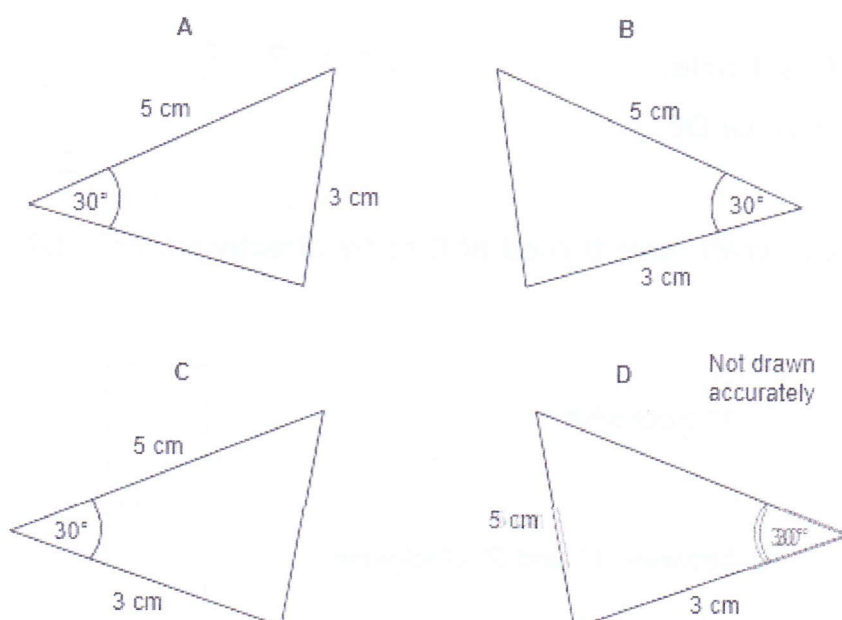
Prove that triangle ABE is congruent to triangle DCE.

$BE = CE$ (Equilateral $\triangle$)
 $AE = ED$ (E midpoint of AD)
 $\angle AEB = \angle DEC$ (alternate angles)

SAS

[4]

19. Here are four triangles.



a) Which two triangles are congruent? Circle your answers.

A

B

C

D

[1]

b) Circle the reason for your answer to part (a).

SSS

ASA

SAS

RHS

[1]

CREDITS AND NOTES

Q	Awarding Body	Q	Awarding Body	Q	Awarding Body
1	WJEC Eduqas	8	AQA	15	Pearson Edexcel
2	AQA	9	Pearson Edexcel	16	Pearson Edexcel
3	AQA	10	AQA	17	AQA
4	Pearson Edexcel	11	OCR	18	OCR
5	OCR	12	OCR	19	AQA
6	OCR	13	Pearson Edexcel		
7	Pearson Edexcel	14	Pearson Edexcel		

Notes:

These questions have been retyped from the original sample/specimen assessment materials and whilst every effort has been made to ensure there are no errors, any that do appear are mine and not the exam boards (similarly any errors I have corrected from the originals are also my corrections and not theirs!).

Please also note that the layout in terms of fonts, answer lines and space given to each question does not reflect the actual papers to save space.

These questions have been collated by me as the basis for a GCSE working party set up by the GLOW maths hub - if you want to get involved please get in touch. The objective is to provide support to fellow teachers and to give you a flavour of how different topics "could" be examined. They should not be used to form a decision as to which board to use. There is no guarantee that a topic will or won't appear in the "live" papers from a specific exam board or that examination of a topic will be as shown in these questions.



Links:

AQA <http://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300>

OCR <http://ocr.org.uk/gcsemaths>

Pearson Edexcel <http://qualifications.pearson.com/en/qualifications/edexcel-gcses/mathematics-2015.html>

WJEC Eduqas <http://www.eduqas.co.uk/qualifications/mathematics/gcse/>

Contents:

This version contains questions from:

AQA – Sample Assessment Material, Practice set 1 and Practice set 2

OCR – Sample Assessment Material and Practice set 1

Pearson Edexcel – Sample Assessment Material, Specimen set 1 and Specimen set 2

WJEC Eduqas – Sample Assessment Material