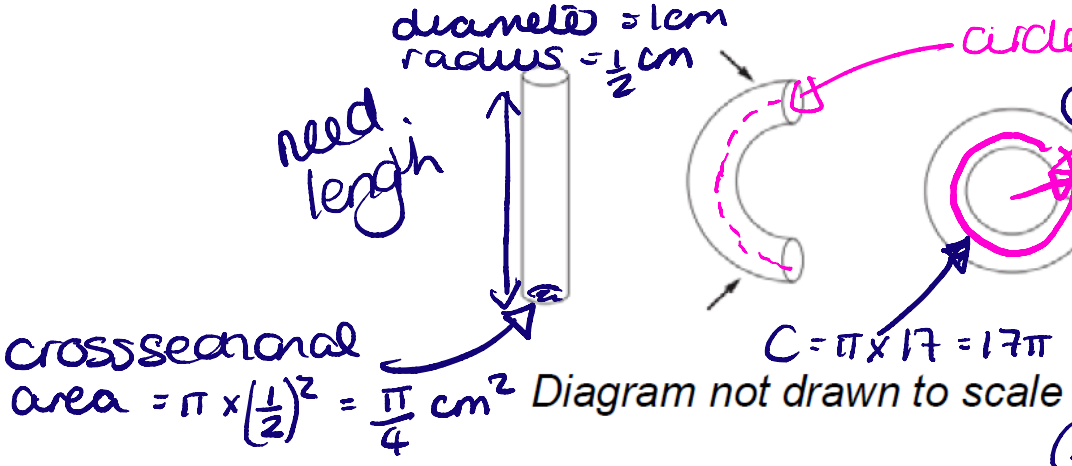
Volume of Prisms, Cones, Pyramids & Spheres (H)

A collection of 9-1 Maths GCSE Sample and Specimen questions from AQA, OCR, Pearson-Edexcel and WJEC Eduqas.

Name:	Mel@JustMaths
Total Marks:	

1. A cylinder is made of bendable plastic.
   A dog's toy is made by bending the cylinder to form a ring.



① assuming a constant cross sectional area with a radius of 8.5cm so diameter = 17cm

The inner radius of the dog's toy is 8 cm.
The outer radius of the dog's toy is 9 cm.

② assuming height of cylinder is the same as the circumference of the middle of the ring.

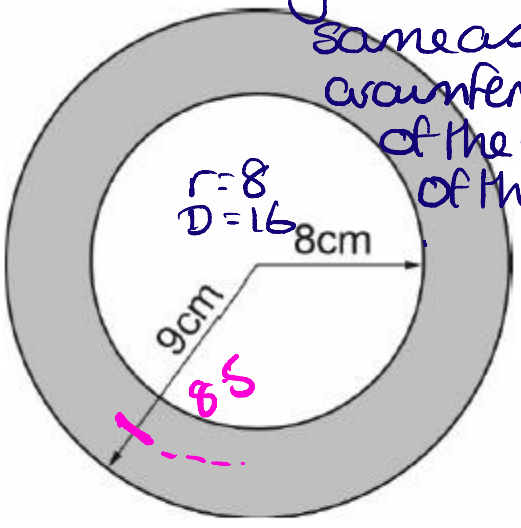


Diagram not drawn to scale

Calculate an approximate value for the volume of the dog's toy.

State and justify what assumptions you have made in your calculations and the impact they have had on your results.

Volume = $\frac{\pi}{4} \times 17\pi$

= 41.9458187 cm^3

If I'd used a smaller radius/diameter for the height the volume would be smaller. Bigger radius/diameter would result in a bigger volume.

[7]

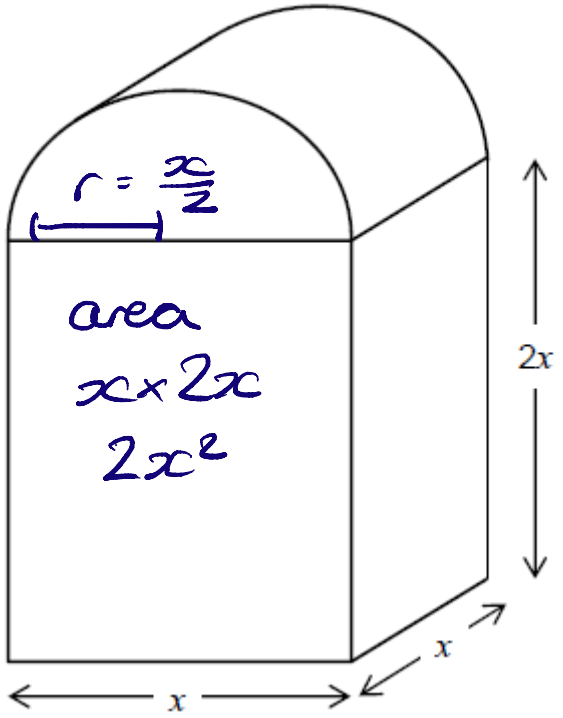
$A = \pi r^2$

$C = \pi D$

2. In this question all dimensions are in centimetres.

A solid has uniform cross section.

The cross section is a rectangle and a semicircle joined together.



semicircle area $= \frac{1}{2} \times \pi \times \left(\frac{x}{2}\right)^2$

$= \frac{\pi}{2} \times \frac{x^2}{4} = \frac{\pi x^2}{8}$

Volume $= \frac{\pi x^2}{8} \times x + 2x^2 \times x$

$= 2x^3 + \frac{\pi x^3}{8}$

Work out an expression, in cm^3, for the total volume of the solid.

Write your expression in the form $ax^3 + \frac{1}{b}\pi x^3$ where a and b are integers.

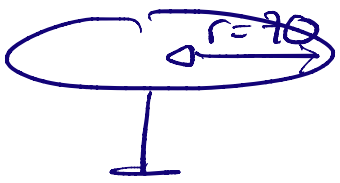
$V = 2x^3 + \frac{1}{8}\pi x^3$

$a = 2 \quad b = 8$

[4]

3. A circular table top has radius 70 cm.

(a) Calculate the area of the table top in cm^2, giving your answer as a multiple of π.

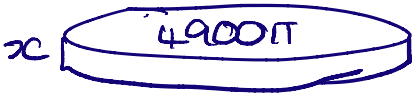


area $= \pi \times 70^2$

(a) ..4900π..... cm^2 [2]

(b) The volume of the table top is 17 150π cm^3.

Calculate the thickness of the table top.



$17,150 = 4900\pi \times x$

$x = \frac{17150\pi}{4900\pi}$

(b) ..3.5..... cm [2]

4. The volume of Earth is 1.08×10^{12} km^3.

   The volume of Jupiter is 1.43×10^{15} km^3.

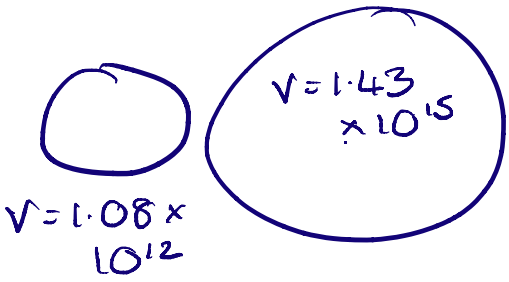
   How many times larger is the radius of Jupiter than the radius of Earth?

   Assume that Jupiter and Earth are both spheres.

   The volume v of a sphere with radius r is $V = \frac{4}{3}\pi r^3$

$r^3 = 1.43 \times 10^{15} \times \frac{3}{4\pi}$

$r = 69890.12374$



$1.08 \times 10^{12} = \frac{4}{3}\pi r^3$

$r^3 = 1.08 \times 10^{12} \times \frac{3}{4\pi}$

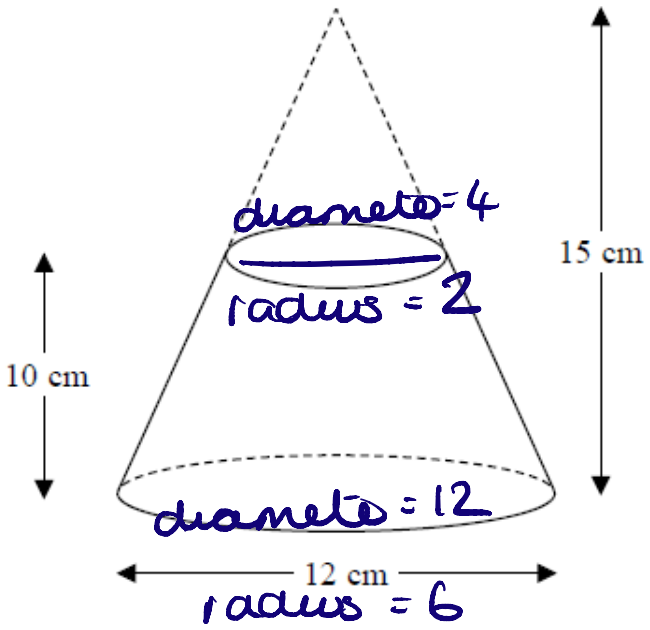
$r = 6364.706508$

$69890.12374 \div 6364.70650 = 10.98088\ldots$

$\simeq$ 11 times bigger.

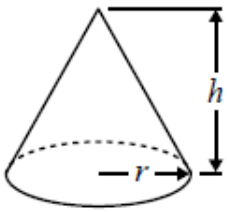
..... [4]

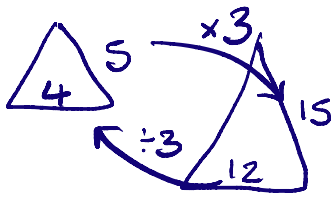
5. A frustum is made by removing a small cone from a large cone as shown in the diagram.



$D = \frac{M}{V}$ $M = D \times V$

Volume of cone $= \frac{1}{3}\pi r^2 h$





The frustum is made from glass.
The glass has a density of 2.5 g / cm^3

Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

$V_{SMALL} = \frac{1}{3}\pi \times 2^2 \times 5 = \frac{20}{3}\pi$

$V_{LARGE} = \frac{1}{3}\pi \times 6^2 \times 15 = 180\pi$

Frustum volume $= 180\pi - \frac{20}{3}\pi$

$= 544.5427 \text{cm}^3$

MASS $= 544.54 \times 2.5 = 1361.3568$g

1361 .....g [4]

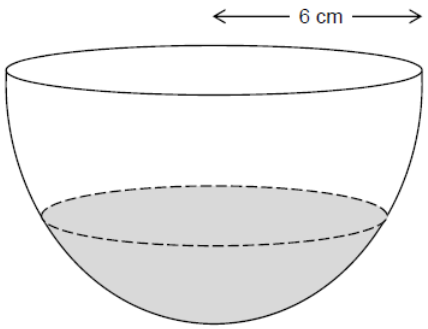
6. A bowl is a hemisphere with radius 6 cm

Water fills two-fifths of the volume of the bowl.

hemisphere

$\frac{1}{2} \times \frac{4}{3} \times \pi \times 6^2$

$= 144\pi$



The water is poured into a hollow cone.

The depth of the water in the cone is 12 cm

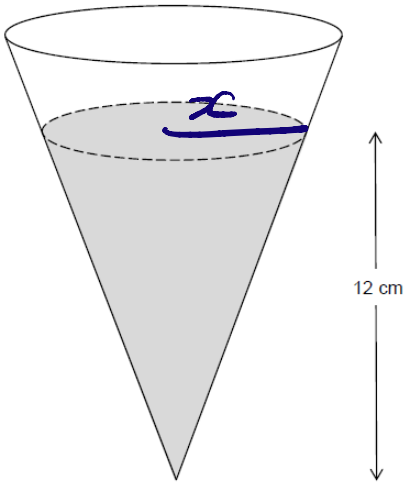
$\frac{2}{5} 144\pi = \frac{1}{3} \pi x^2 \times 12$

$x^2 = \frac{288 \times 3}{60}$

$x^2 = 14.4$

$x = \sqrt{14.4}$

$x = 3.7947733$



$x = 3.8\text{cm}$ (1dp)

Volume of a sphere $= \frac{4}{3}\pi r^3$	where r is the radius.
Volume of a cone $= \frac{1}{3}\pi r^2 h$	where r is the radius and h is the perpendicular height

Work out the radius of the surface of the water in the cone.

[4]

$1000cm^3 = 1$ litre.

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7. The diagram shows a sand pit.

The sand pit is in the shape of a cuboid.

Sally wants to fill the sand pit with sand.

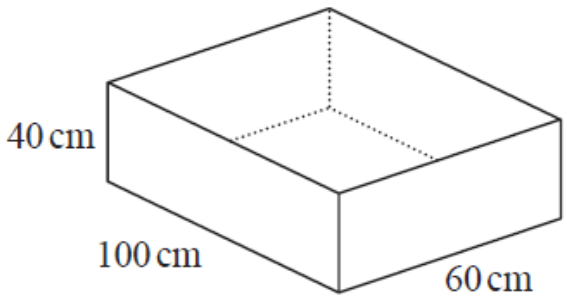
A bag of sand costs £2.50

There are 8 litres of sand in each bag.

Sally says,

"The sand will cost less than £70"

Show that Sally is wrong.



Volume = $40 \times 100 \times 60$
$= 240000 cm^3$

240 litres

1 bag = 8 litres
30 bags. 240 litres ↓ ×30

$30 \times 2.50 = £75$. It will cost 75 which shows Sally is wrong

[5]

8. The diagram shows an oil tank in the shape of a prism.

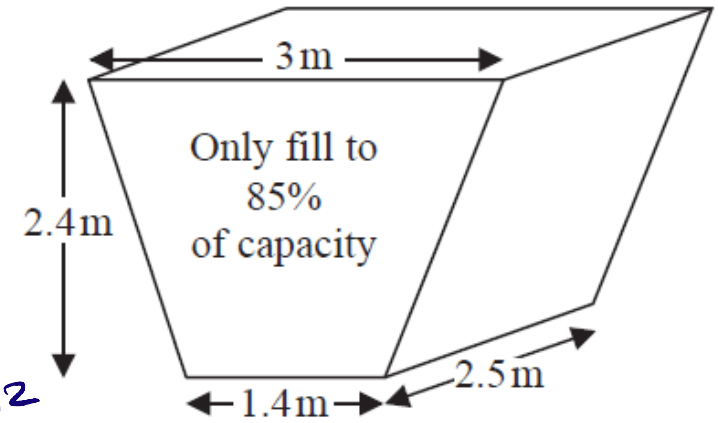
The cross section of the prism is a trapezium.

The tank is empty.

Oil flows into the tank.

After one minute there are 300 litres of oil in the tank.

Assume that oil continues to flow into the tank at this rate.



area of front face
$= \frac{1}{2}(1.4 + 3) \times 2.4 = 5.28 m^2$

(a) Work out how many more minutes it takes for the tank to be 85% full of oil.

(1 m^3 = 1000 litres)

Volume = $5.28 \times 2.5 = 13.2 m^3$ = 13,200 litres when full.
85% of capacity = 11220 litres.
Difference = 10920 to get to 85%. → assuming 300 litres/min
$10920 \div 300$

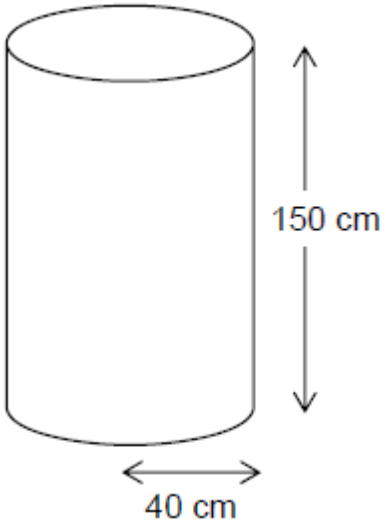
36.4 ..... minutes [5]

The assumption about the rate of flow of the oil could be wrong.

(b) Explain how this could affect your answer to part (a).

I have assumed a constant flow of 300 litres per min.
Slower flow means it'll take longer. Faster flow will be quicker to fill [1]

9. A water tank is a cylinder with radius 40 cm and depth 150 cm



area = $\pi \times 40^2$
= 1600π cm^2

Volume = $1600\pi \times 150$
= $240{,}000\pi$ cm^3
= 753,982.24cm^3

It is filled at the rate of 0.2 litres per second.

1 litre = 1000 cm^3

Does it take longer than 1 hour to fill the tank? 1hour = 60 × 60 = 3600 seconds

You must show your working.

Volume = 753.982 litres

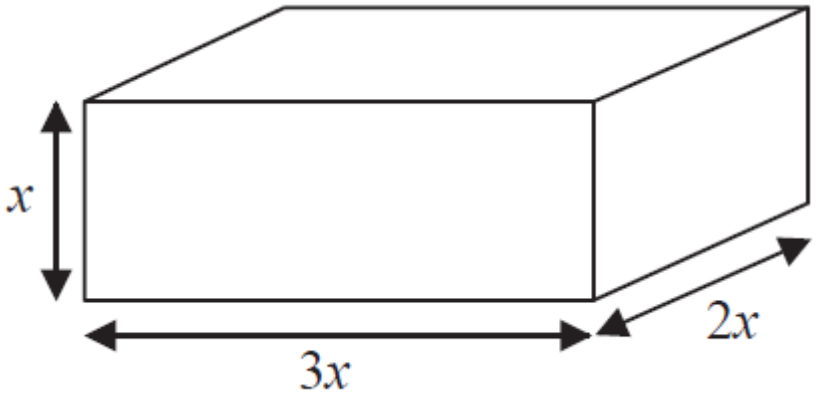
Time to fill = 3769.91 seconds

yes. it takes longer

3769.91 > 3600

[4]

10. Here is a cuboid.



All measurements are in centimetres.

x is an integer.

The total volume of the cuboid is less than 900 cm^3

Show that $x \leq 5$

$V = 3x \times x \times 2x$

$= 6x^3$

$6x^3 < 900$

$x^3 < \frac{900}{6}$

$x^3 < 150$

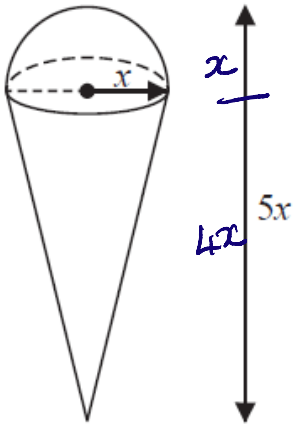
$x < \sqrt[3]{150}$

$x < 5.313\text{cm}$

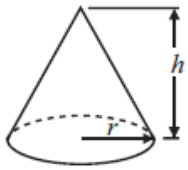
x must be an integer

$x \leq 5$ [3]

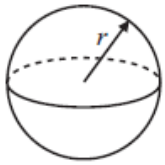
11. A solid is made by putting a hemisphere on top of a cone.



Volume of cone $= \frac{1}{3}\pi r^2 h$



Volume of sphere $= \frac{4}{3}\pi r^3$



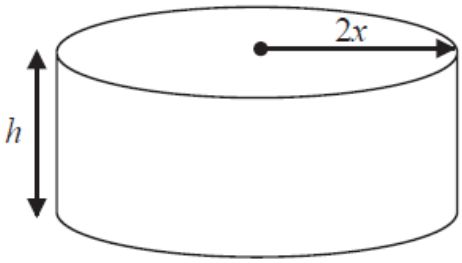
The total height of the solid is 5x
The radius of the base of the cone is x
The radius of the hemisphere is x

A cylinder has the same volume as the solid.
The cylinder has radius 2x and height h
All measurements are in centimetres.

$V = \frac{1}{2}\frac{4}{3} \times \pi \times x^3 + \frac{1}{3}\pi \times x^2 \times 4x$

$= \frac{4}{6}\pi x^3 + \frac{4}{3}\pi x^3$

$= 2\pi x^3$



$\pi \times 4x^2 \times h$

$4\pi x^2 h = 2\pi x^3$

$h = \frac{2\pi x^3}{4\pi x^2}$

$= \frac{x}{2}$

$h = \frac{x}{2}$

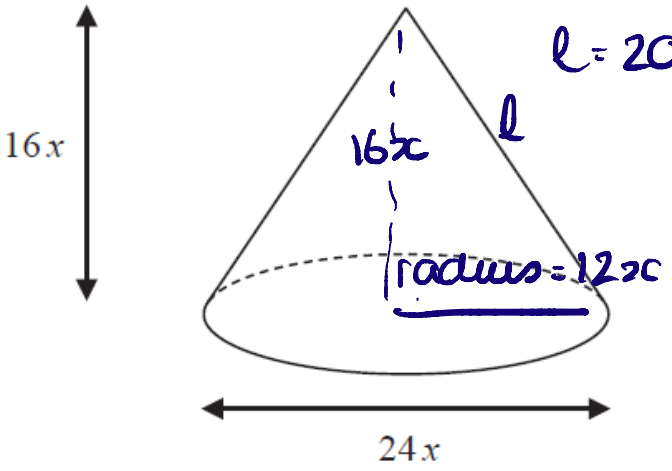
Find a formula for h in terms of x
Give your answer in its simplest form.

[5]

12. The diagram shows a solid cone.

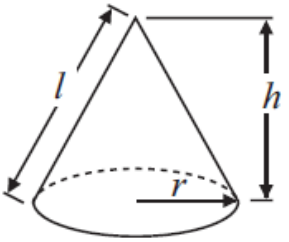
$l^2 = (16x)^2 + (12x)^2 = 400x^2$

$l = 20x$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



The diameter of the base of the cone is $24x$ cm.
The height of the cone is $16x$ cm.

The curved surface area of the cone is 2160π cm^2.
The volume of the cone is $V\pi$ cm^3, where V is an integer.

Find the value of V.

$2160\pi = \pi \times 12x \times 20x$

$2160 = 240x^2$

$x = \sqrt{\frac{2160}{240}}$

$= 3$

so radius $= 12 \times 3 = 36$

height $= 16 \times 3 = 48$

$Vol = \frac{1}{3} \times \pi \times 36^2 \times 48$

$= 20736\pi \text{ cm}^3$

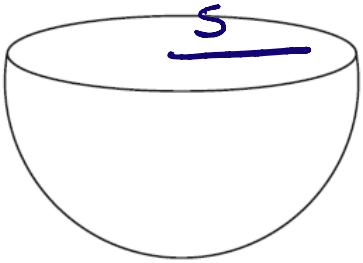
$\therefore V = 20736$

[5]

13. The diagram shows a solid hemisphere.

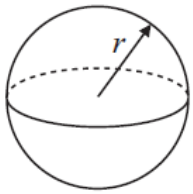
(2)

Area = $\pi \times 5^2$
$= 25\pi$



Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



(1)

The volume of the hemisphere is $\frac{250}{3}\pi$ $= \frac{1}{2} \times \frac{4}{3}\pi r^3$ $r^3 = \frac{250}{3} \times \frac{6}{4} \times \frac{\pi}{\pi}$

Work out the exact total surface area of the solid hemisphere.
Give your answer as a multiple of π.

$r = \sqrt[3]{125}$
$= 5$

Surface area of hemisphere $= \frac{1}{2} \times 4\pi \times 5^2 = 50\pi$

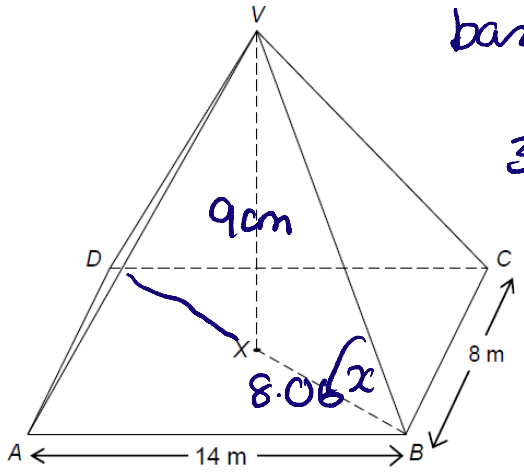
Total $= 25\pi + 50\pi$

75π ..... cm^2 [4]

14. Volume of a pyramid = $\frac{1}{3}$ x area of base x perpendicular height

VABCD is a rectangular-based pyramid with volume 336 m^3

X is the centre of the horizontal base, directly below *V*.



base area $= 8 \times 14 = 112 cm^2$

$336 = \frac{1}{3} \times 112 \times h$

$h = \frac{336 \times 3}{112}$
$= 9cm$

$DB^2 = 14^2 + 8^2$
$= 196 + 64$
$= 260$
$DB = \sqrt{260}$

Work out the angle between *VB* and the base.

$BX = \frac{\sqrt{260}}{2}$
$= 8.0622 cm$

$\tan x = \frac{9}{8.0622}$ $x = \tan^{-1}\frac{9}{8.0622}$

$x = 48.15^\circ$

[6]

15. A solid metal sphere has radius 9.8 cm.

The metal has a density of 5.023 g/cm^3.

Lynne estimates the mass of this sphere to be 20 kg.

Show that this is a reasonable estimate for the mass of the sphere.

[The volume V of a sphere with radius r is V = $\frac{4}{3}\pi r^3$]

$V = \frac{4}{3} \pi \times 9.8^3 = 3942.4558\ldots \text{cm}^3$

$D = \frac{m}{V}$ ✓✓ $\quad \text{mass} = 5.023 \times 3942.45 = 19802.955\ldots \text{g}$

$\approx 20\text{kg}$ [5]

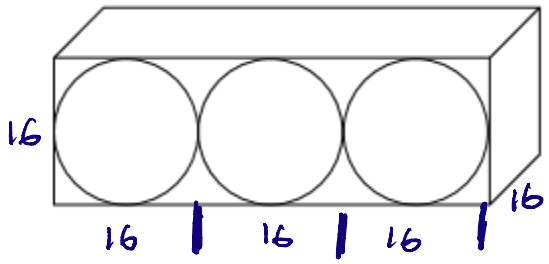
16. Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius.

a) Work out the volume of a sphere of radius 8 cm

$\frac{4}{3} \times \pi \times 8^3 = 2144.66 \text{cm}^3$

[2]

b) Three spheres of radius 8 cm are packed tightly into a cuboid as shown.



Work out the volume of the cuboid.

$48 \times 16 \times 16 = 12288 \text{cm}^3$

[4]

CREDITS AND NOTES

Question	Awarding Body	Question	Awarding Body
1	WJEC Eduqas	9	AQA
2	AQA	10	Pearson Edexcel
3	OCR	11	Pearson Edexcel
4	OCR	12	Pearson Edexcel
5	Pearson Edexcel	13	Pearson Edexcel
6	AQA	14	AQA
7	Pearson Edexcel	15	OCR
8	Pearson Edexcel	16	AQA

Notes:

These questions have been retyped from the original sample/specimen assessment materials and whilst every effort has been made to ensure there are no errors, any that do appear are mine and not the exam board s (similarly any errors I have corrected from the originals are also my corrections and not theirs!).

Please also note that the layout in terms of fonts, answer lines and space given to each question does not reflect the actual papers to save space.

These questions have been collated by me as the basis for a GCSE working party set up by the GLOW maths hub - if you want to get involved please get in touch. The objective is to provide support to fellow teachers and to give you a flavour of how different topics "could" be examined. They should not be used to form a decision as to which board to use. There is no guarantee that a topic will or won't appear in the "live" papers from a specific exam board or that examination of a topic will be as shown in these questions.

Links:

AQA http://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300

OCR http://ocr.org.uk/gcsemaths

Pearson Edexcel http://qualifications.pearson.com/en/qualifications/edexcel-gcses/mathematics-2015.html

WJEC Eduqas http://www.eduqas.co.uk/qualifications/mathematics/gcse/

Contents:

This version contains questions from:

AQA – Sample Assessment Material, Practice set 1 and Practice set 2

OCR – Sample Assessment Material and Practice set 1

Pearson Edexcel – Sample Assessment Material, Specimen set 1 and Specimen set 2

WJEC Eduqas – Sample Assessment Material