



# Mathematics

## Programme of study for key stage 4

(This is an extract from The National Curriculum 2007)

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## Curriculum aims

Learning and undertaking activities in mathematics contribute to achievement of the curriculum aims for all young people to become:

- successful learners who enjoy learning, make progress and achieve
- confident individuals who are able to live safe, healthy and fulfilling lives
- responsible citizens who make a positive contribution to society.

## The importance of mathematics

Mathematical thinking is important for all members of a modern society as a habit of mind for its use in the workplace, business and finance, and for personal decision-making. Mathematics is fundamental to national prosperity in providing tools for understanding science, engineering, technology and economics. It is essential in public decision-making and for participation in the knowledge economy.

Mathematics equips students with uniquely powerful ways to describe, analyse and change the world. It can stimulate moments of pleasure and wonder for all students when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections. Students who are functional in mathematics and financially capable are able to think independently in applied and abstract ways, and can reason, solve problems and assess risk.

Mathematics is a creative discipline. The language of mathematics is international. The subject transcends cultural boundaries and its importance is universally recognised. Mathematics has developed over time as a means of solving problems and also for its own sake.



# 1 Key concepts

There are a number of key concepts that underpin the study of mathematics. Students need to understand these concepts in order to deepen and broaden their knowledge, skills and understanding.

## 1.1 Competence

- a Applying suitable mathematics accurately within the classroom and beyond.
- b Communicating mathematics effectively.
- c Selecting appropriate mathematical tools and methods, including ICT.

## 1.2 Creativity

- a Combining understanding, experiences, imagination and reasoning to construct new knowledge.
- b Using existing mathematical knowledge to create solutions to unfamiliar problems.
- c Posing questions and developing convincing arguments.

### EXPLANATORY NOTES

**Study of mathematics:** This is concerned with the learning processes for mathematics.

**Applying suitable mathematics:** This requires fluency and confidence in a range of mathematical techniques and processes that can be applied in a widening range of familiar and unfamiliar contexts, including managing money, assessing risk, problem-solving and decision-making.

**Communicating mathematics:** Students should be familiar with and confident about mathematical notation and conventions and be able to select the most appropriate way to communicate mathematics, both orally and in writing. They should also be able to understand and interpret mathematics presented in a range of forms.

**Selecting appropriate mathematical tools:** Students should be familiar with a range of resources and tools, including graphical calculators, dynamic geometry and spreadsheets, which can be used to work on mathematics.

**Mathematical methods:** At the heart of mathematics are the concepts of equivalence, proportional thinking, algebraic structure, relationships, axiomatic systems, symbolic representation, proof, operations and their inverses.

**Posing questions:** This involves students adopting a questioning approach to mathematical activity, asking questions such as 'How true?' and 'What if...?'

### 1.3 Applications and implications of mathematics

- a Knowing that mathematics is a rigorous, coherent discipline.
- b Understanding that mathematics is used as a tool in a wide range of contexts.
- c Recognising the rich historical and cultural roots of mathematics.
- d Engaging in mathematics as an interesting and worthwhile activity.

### 1.4 Critical understanding

- a Knowing that mathematics is essentially abstract and can be used to model, interpret or represent situations.
- b Recognising the limitations and scope of a model or representation.



#### EXPLANATORY NOTES

**Mathematics is used as a tool:** This includes using mathematics as a tool for making financial decisions in personal life and for solving problems in fields such as building, plumbing, engineering or geography. Current applications of mathematics in everyday life include internet security, weather forecasting, modelling changes in society and the environment, and managing risk (eg insurance, investments and pensions). Mathematics can be used as a way of perceiving the world, for example the symmetry in architecture and nature and the geometry of clothing.

**Historical and cultural roots of mathematics:** Mathematics has a rich and fascinating history and has been developed across the world to solve problems and for its own sake. Students should learn about problems from the past that led to the development of particular areas of mathematics, appreciate that pure mathematical findings sometimes precede practical applications, and understand that mathematics continues to develop and evolve.

**Limitations and scope:** Mathematics equips students with the tools to model and understand the world around them. This enables them to engage with complex issues, such as those involving financial capability or environmental dilemmas. For example, mathematical skills are needed to compare different methods of borrowing and paying back money, but the final decision may include other dimensions, such as comparing the merits of using a credit card that promotes a particular charity with one offering the lowest overall cost. The mathematical model or representation may have properties that are not relevant to the situation.

## 2 Key processes

These are the essential skills and processes in mathematics that students need to learn to make progress.

### 2.1 Representing

Students should be able to:

- a identify the mathematical aspects of the situation or problem
- b compare and evaluate representations of a situation before making a choice
- c simplify the situation or problem in order to represent it mathematically using appropriate variables, symbols, diagrams and models
- d select mathematical information, methods, tools and models to use.

### 2.2 Analysing

#### Use mathematical reasoning

Students should be able to:

- a make connections within mathematics
- b use knowledge of related problems
- c visualise and work with dynamic images
- d identify and classify patterns
- e make and justify conjectures and generalisations, considering special cases and counter-examples

#### EXPLANATORY NOTES

**Processes in mathematics:** The key processes in this section are clearly related to the different stages of problem-solving and the handling data cycle.

**Representing:** Representing a situation places it into the mathematical form that will enable it to be worked on. Students should explore mathematical situations independently; identify the major mathematical features of a problem and potentially fruitful paths; use and amend representations in the light of experience; identify what has been included and what has been omitted; and break the problem down (eg starting with a simple case, working systematically through cases, identifying different components that need to be brought together and identifying the stages in the solution process).

**Identify:** This includes identifying questions that can be addressed using statistical methods.

**Representations of a situation:** This includes moving between different representations in pure and applied contexts, for example in an engineering context or assembling a piece of flat-pack furniture.

**Simplify:** This involves using and constructing models with increasing sophistication and understanding the constraints that are being introduced.

**Select mathematical information, methods, tools and models:** This involves examining a situation systematically, identifying different ways of breaking a task down and identifying gaps in personal knowledge. In statistical investigations it includes planning to minimise sources of bias when conducting experiments and surveys and using a wide variety of methods for collecting primary and secondary data. ICT tools can be used for mathematical applications, including iteration and algorithms.

**Make connections:** For example, realising that an equation, a table of values and a line on a graph can all represent the same thing, or understanding that an intersection between two lines on a graph can represent the solution to a problem.

**Generalisations:** Students should represent generalisations covering a range of mathematical content and contexts in different ways (including algebra).

- f explore the effects of varying values and look for invariance and covariance
- g take account of feedback and learn from mistakes
- h work logically towards results and solutions, recognising the impact of constraints and assumptions
- i identify a range of techniques that could be used to tackle a problem, appreciating that more than one approach may be necessary
- j reason inductively, deduce and prove.

### Use appropriate mathematical procedures

Students should be able to:

- k make accurate mathematical diagrams, graphs and constructions on paper and on screen
- l calculate accurately, using mental methods or calculating devices as appropriate
- m manipulate numbers, algebraic expressions and equations and apply routine algorithms
- n use accurate notation, including correct syntax when using ICT
- o record methods, solutions and conclusions
- p estimate, approximate and check working.

**Mathematical thinking is important for all members of a modern society as a habit for its use in the workplace, business and finance, and for personal decision-making**

#### EXPLANATORY NOTES

**Varying values:** This involves identifying variables and controlling these to explore a situation. ICT could be used to explore many cases, including statistical situations with underlying random or systematic variation.

**A range of techniques:** For example, working backwards, looking at simpler cases, choosing one or more of a numerical, analytical or graphical approach, and being able to use techniques independently.

**Tackle a problem:** This includes using mathematical reasoning to explain and justify inferences when analysing data.

**Reason inductively:** This involves using particular examples to suggest a general statement.

**Deduce:** This involves using reasoned arguments to derive or draw a conclusion from something already known.

**Mathematical procedures:** This includes procedures for collecting, processing and representing data.

**Calculating devices as appropriate:** For example, when calculation without a calculator will take an inappropriate amount of time.

**Record methods:** This involves increasing use of more formal methods, including algebra, and more formal proofs.

## 2.3 Interpreting and evaluating

Students should be able to:

- a form convincing arguments to justify findings and general statements
- b consider the assumptions made and the appropriateness and accuracy of results and conclusions
- c appreciate the strength of empirical evidence and distinguish between evidence and proof
- d look at data to find patterns and exceptions
- e relate their findings to the original question or conjecture, and indicate reliability
- f make sense of someone else's findings and judge their value in the light of the evidence they present
- g critically examine strategies adopted.

## 2.4 Communicating and reflecting

Students should be able to:

- a use a range of forms to communicate findings to different audiences
- b engage in mathematical discussion of results
- c consider the elegance and efficiency of alternative solutions
- d look for equivalence in relation to both the different approaches to the problem and different problems with similar structures
- e give examples of similar contexts they have previously encountered and identify how these contexts differed from or were similar to the current situation and how and why the same, or different, strategies were used.

### EXPLANATORY NOTES

**Interpreting:** This includes interpreting data and involves looking at the results of an analysis and deciding how the results relate to the original problem.

**Form convincing arguments:** This involves using more formal arguments and proof to support cases and appreciating the difference between inductive and deductive arguments.

**Strength of empirical evidence:** This includes evidence gathered when using ICT to explore cases and understanding the effects of sample size when interpreting data.

**Patterns and exceptions:** Students should understand that random processes are unpredictable.

**Light of the evidence:** Students may find, for example, errors in an argument or missing steps or exceptions to a given case. This includes interpreting information presented by the media and through advertising.

**Critically examine strategies:** This includes examining elegance of approach and the strength of evidence in their own or other people's arguments.

**Communicating and reflecting:** This involves communicating findings to others and reflecting on different approaches.

**Range of forms:** This includes appropriate language (both written and verbal forms), suitable graphs and diagrams, standard notation and labelling conventions and ICT models.

**Alternative solutions:** These include multiple approaches and solutions using ICT.

## 3 Range and content

This section outlines the breadth of the subject on which teachers should draw when teaching the key concepts and key processes.

The study of mathematics should enable students to apply their knowledge, skills and understanding to relevant real-world situations.

The study of mathematics should include:

### 3.1 Number and algebra

- a real numbers, their properties and their different representations
- b rules of arithmetic applied to calculations and manipulations with real numbers, including standard index form and surds
- c proportional reasoning, direct and inverse proportion, proportional change and exponential growth
- d upper and lower bounds
- e linear, quadratic and other expressions and equations
- f graphs of exponential and trigonometric functions
- g transformation of functions
- h graphs of simple loci

### 3.2 Geometry and measures

- a properties and mensuration of 2D and 3D shapes
- b circle theorems
- c trigonometrical relationships
- d properties and combinations of transformations
- e 3D coordinate systems
- f vectors in two dimensions
- g conversions between measures and compound measures

#### EXPLANATORY NOTES

**Relevant real-world situations:** Mathematical skills are required in many workplace settings, for example understanding relationships between variables in stock control (food processing) or calculating and monitoring quantifiable variables of a hotel's performance (tourism).

**Rules of arithmetic:** This includes knowledge of operations and inverse operations and how calculators use precedence. Students should understand that not all calculators use algebraic logic and may give different answers for calculations such as  $1 + 2 \times 3$ .

**Calculations and manipulations with real numbers:** This includes using mental and written methods to make sense of everyday situations such as temperature, altitude, financial statements and transactions.

**Proportion:** This includes percentages and applying concepts of ratio and proportion to contexts such as value for money, scales, plans and maps, cooking and statistical information (eg 9 out of 10 people prefer...).

**Linear, quadratic and other expressions and equations:** This includes relationships between solutions found using algebraic or graphical representations and trial and improvement methods. Simultaneous equations should include one linear and one quadratic equation.

**Properties:** This includes using properties to make accurate constructions of 2D and 3D shapes.

**Mensuration of 2D and 3D shapes:** This includes area of a triangle as  $\frac{1}{2}ab\sin C$ , cones, pyramids and spheres.

#### Transformations

This includes enlargements with negative scale factors and transformations using ICT.



### 3.3 Statistics

- a the handling data cycle
- b presentation and analysis of large sets of grouped and ungrouped data, including box plots and histograms, lines of best fit and their interpretation
- c measures of central tendency and spread
- d experimental and theoretical probabilities of single and combined events.

#### EXPLANATORY NOTES

**The handling data cycle:** This is closely linked to the mathematical key processes and consists of:

- specifying the problem and planning (representing)
- collecting data (representing and analysing)
- processing and presenting the data (analysing)
- interpreting and discussing the results (interpreting and evaluating).

**Presentation and analysis:** This includes the use of ICT.

**Grouped and ungrouped data:** This includes the use of two-way tables.

**Measures of central tendency and spread:** This includes using measures of average and range to compare distributions.

**Probabilities:** This includes applying ideas of probability and risk to gambling, safety issues and the financial services sector, and simulations using ICT to represent a probability experiment, such as rolling two dice and adding the scores.

**Combined events:** This includes systematic approaches to listing all outcomes.

## 4 Curriculum opportunities

During the key stage students should be offered the following opportunities that are integral to their learning and enhance their engagement with the concepts, processes and content of the subject.

The curriculum should provide opportunities for students to:

- a develop confidence in an increasing range of methods and techniques
- b work on sequences of tasks that involve using the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts
- c work on open and closed tasks in a variety of real and abstract contexts that allow them to select the mathematics to use
- d work on problems that arise in other subjects and in contexts beyond the school
- e work on tasks that bring together different aspects of concepts, processes and mathematical content
- f work collaboratively as well as independently in a range of contexts
- g become familiar with a range of resources, including ICT, so that they can select appropriately.



### EXPLANATORY NOTES

**Other subjects:** For example, representing and analysing data in geography, using formulas and relationships in science, understanding number structure and currency exchange in modern foreign languages, measuring and making accurate constructions in design and technology, and managing money in economic wellbeing and financial capability.

**Contexts beyond the school:** For example, conducting a survey into consumer habits; using formulas; planning a holiday budget; designing a product; and measuring for home improvements. Mathematical skills contribute to financial capability and to other aspects of preparation for adult life.

**Work collaboratively:** This includes talking about mathematics, evaluating their own and others' work and responding constructively, problem-solving in pairs or small groups, and presenting ideas to a wider group.

**Become familiar with a range of resources:** This includes using practical resources and ICT, such as spreadsheets, dynamic geometry, graphing software and calculators, to develop mathematical ideas.