

GCSE Mathematics Specification (8300/3H)

Paper 3 Higher tier

H

Date

Morning

1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Just Maths
Solutions 😊

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the bottom of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book.
- In all calculations, show clearly how you work out your answer.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Please write clearly, in block capitals, to allow character computer recognition.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature _____

Answer **all** questions in the spaces provided.

- 1 Work out the square root of 100 million.

Circle your answer.

[1 mark]

1000

10 000

100 000

1 000 000

- 2 $\mathbf{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} 5 - (-2) \\ -2 - 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \end{pmatrix}$$

Circle the vector $\mathbf{a} - \mathbf{b}$

[1 mark]

$\begin{pmatrix} -3 \\ -5 \end{pmatrix}$

$\begin{pmatrix} 7 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 7 \\ -5 \end{pmatrix}$

- 3 Circle the decimal that is closest in value to $\frac{2}{3} = 0.66666\dots$

[1 mark]

0.6

0.66

0.667

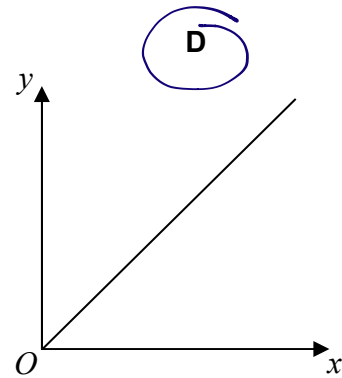
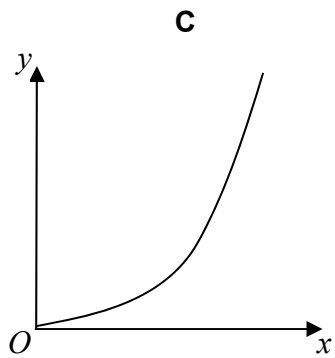
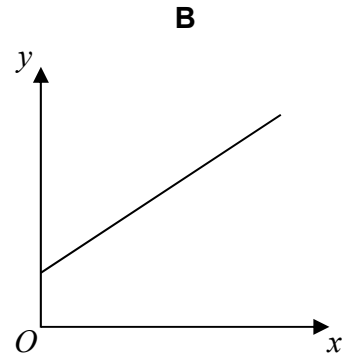
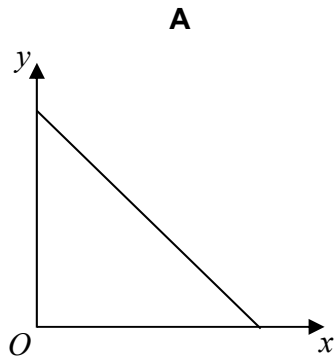
0.67

4 y is directly proportional to x .

Which graph shows this?

Circle the correct letter.

[1 mark]



Turn over for the next question

- 5 In 1999 the minimum wage for adults was £3.60 per hour.
In 2013 it was £6.31 per hour.

Work out the percentage increase in the minimum wage.

[3 marks]

$$6.31 - 3.60 = 2.71 \quad \frac{2.71 \times 100}{3.60}$$

Answer 75.25 %

- 6 A bag contains counters that are red, blue, green or yellow.

| | red | blue | green | yellow |
|--------------------|-----|------|---------|--------|
| Number of counters | 9 | $3x$ | $x - 5$ | $2x$ |

A counter is chosen at random.

The probability it is **red** is $\frac{9}{100}$

Work out the probability it is green.

[4 marks]

$$3x + x - 5 + 2x = 91$$

$$6x = 96$$

$$x = 96/6 = 16$$

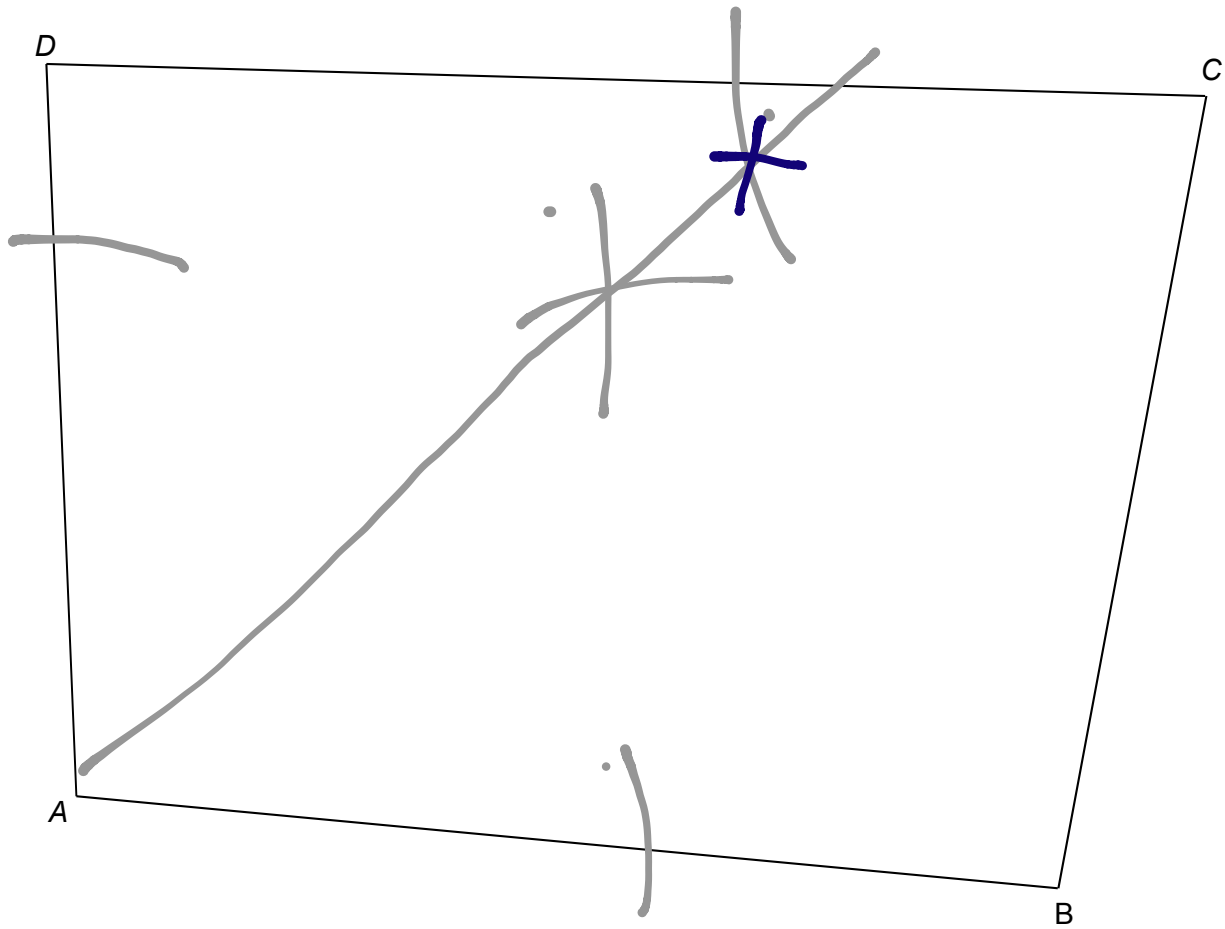
Answer $\frac{11}{100}$

7 Use ruler and compasses to answer this question.

Point P is

- the same distance from AB and AD
- 6 cm from C .

Yours will be more accurate than mine but it'll give you an idea. ... not drawn to scale (f)



Show the position of P on the diagram.

[3 marks]

Turn over for the next question

- 8 (a) Use your calculator to work out $19.42^2 - \sqrt[3]{1006} \div 4.95$

Write down your full calculator display.

[1 mark]

$$\begin{array}{r} 377.1364 - 10.01996013 \div 4.95 \\ \hline 377.1364 - 2.02423437 \end{array}$$

Answer 375.1121656

- 8 (b) Use approximations to check that your answer to part (a) is sensible.

You **must** show your working.

[2 marks]

$$\begin{array}{ll} 19.42^2 \approx 20^2 = 400 & 400 - 10 \div 5 \\ \sqrt[3]{1006} \approx \sqrt[3]{1000} = 10 & 400 - 2 = 398 \\ 4.95 \approx 5 & \end{array}$$

- 9 The exterior angle of a regular polygon is 45°

Circle the name of the regular polygon.

[1 mark]

pentagon

hexagon

octagon

decagon

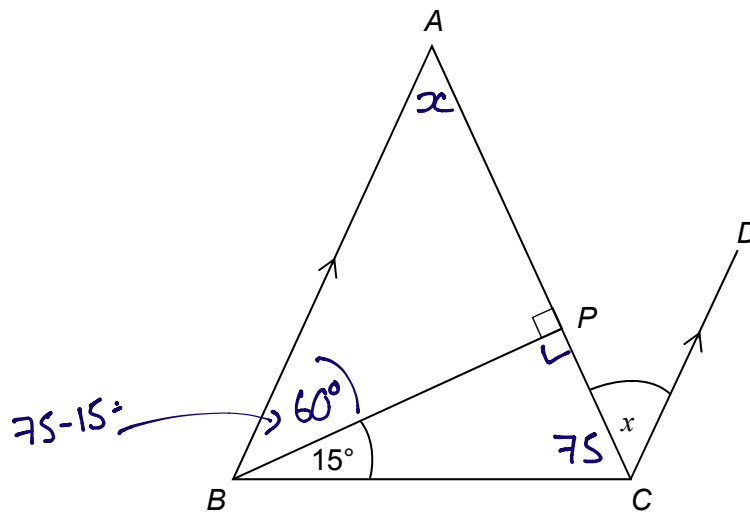
$$360 \div 45 = 8 \text{ sides}$$

10

ABC is a triangle with $AB = AC$

BA is parallel to CD .

Not drawn
accurately



Show that angle $x = 30^\circ$

[3 marks]

$$90 + 15 = 105 \quad 180 - 105 = 75$$

$$\angle ABC = \angle PCB = 75$$

$$\begin{aligned} \angle BAP &= 180 - (90 + 60) \\ &= 30^\circ \end{aligned}$$

11 The pressure at sea level is 101 325 Pascals.

Any rise of 1 km above sea level decreases the pressure by 14%

For example,

at 3 km above sea level the pressure is 14% less than at 2 km

Work out the pressure at 4 km above sea level.

Give your answer to 2 significant figures.

[4 marks]

$$0 = 101325$$

$$1\text{km} = \times 0.86 = 87139.5$$

$$2\text{km} \times 0.86 = 74939.97$$

$$3\text{km} \times 0.86 = 64448.3742$$

$$4\text{km} \times 0.86 = 55425.60181$$

Answer 55,000 Pascals

- 12** Tick whether each statement is true or false.
Give a reason for your answer.

12 (a) When $x^2 = 16$ the **only** value that x can be is 4

[1 mark]

True ☐ False ☒

Reason could also be -4

12 (b) When n is a positive integer, the value of $2n$ is **always** a factor of the value of $20n$.

[1 mark]

True ☒ False ☐

Reason $20n = 2n \times 10$

12 (c) When y is positive, the value of y^2 is **always** greater than the value of y .

[1 mark]

True ☐ False ☒

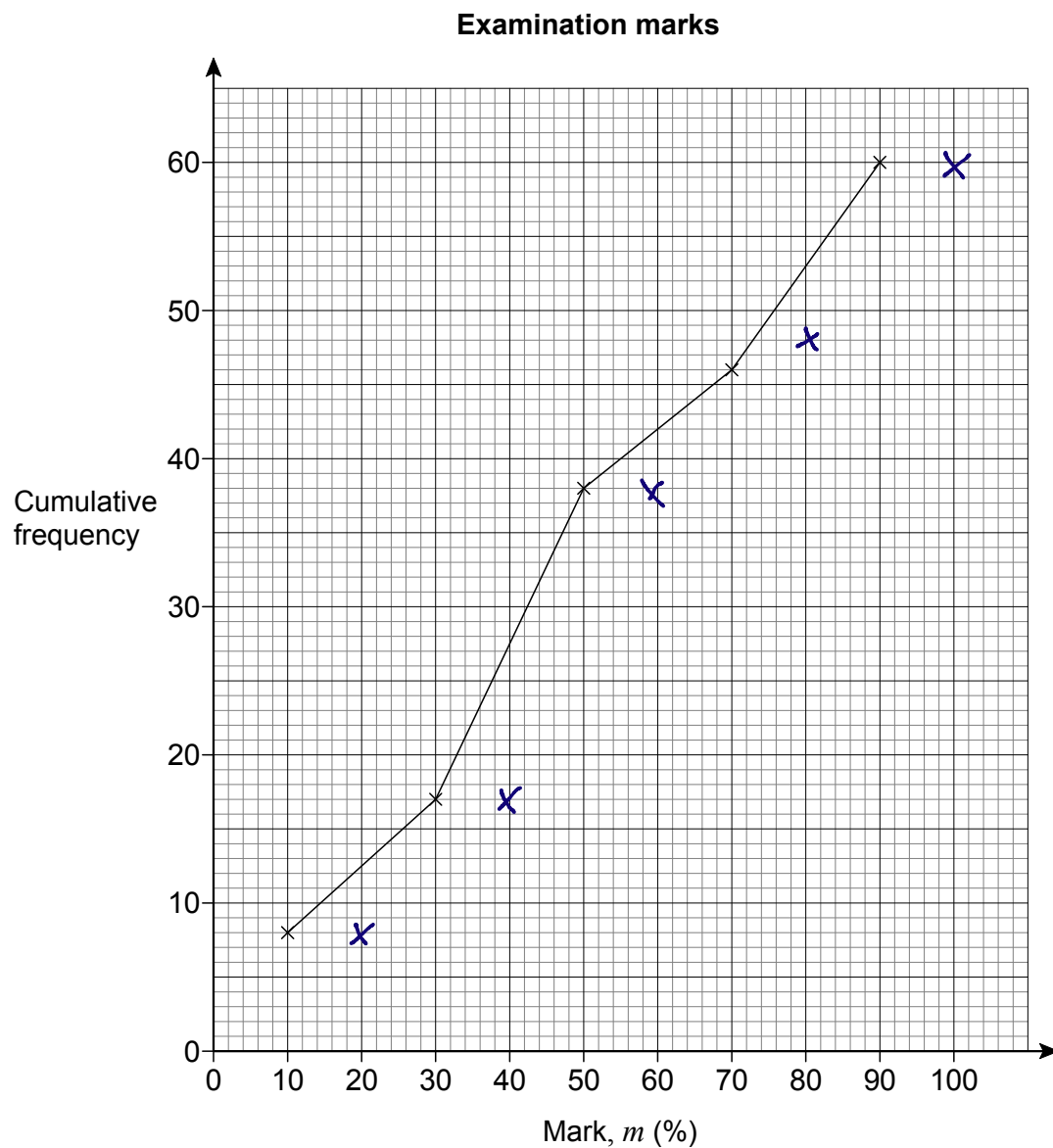
Reason unless y is a fraction < 1
 $\frac{1}{2}^2 = \frac{1}{4}$

13

Here are the examination marks for 60 pupils.

| Mark, m (%) | Frequency | |
|-------------------|-----------|----|
| $0 \leq m < 20$ | 8 | |
| $20 \leq m < 40$ | 9 | 17 |
| $40 \leq m < 60$ | 21 | 38 |
| $60 \leq m < 80$ | 10 | 48 |
| $80 \leq m < 100$ | 12 | 60 |

Molly drew this cumulative frequency graph to show the data.



Make **two** criticisms of Molly's graph.

[2 marks]

Criticism 1 she hasn't plotted the endpoints of each interval

Criticism 2 The CF for the mark at 70 is plotted at 46 not 48

Turn over for the next question

- 14 (a)** The n th term of a sequence is $2^n + 2^{n-1}$

Work out the 10th term of the sequence.

[1 mark]

$$2^{10} + 2^9 =$$

—

Answer 1536

- 14 (b)** The n th term of a different sequence is $4(2^n + 2^{n-1})$

Circle the expression that is equivalent to $4(2^n + 2^{n-1})$

[1 mark]

$$2^{n+2} + 2^{n+1}$$

$$2^{2n} + 2^{2(n-1)}$$

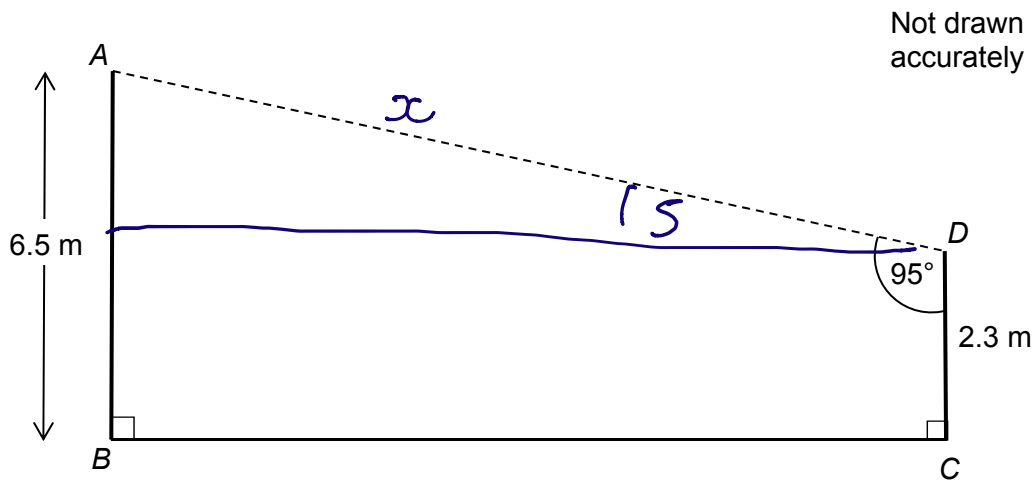
$$8^n + 8^{n-1}$$

$$2^{n+2} + 2^{n-1}$$

15

The diagram shows a design for a zipwire.

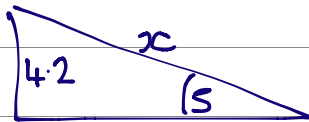
The zipwire will run between the top of two vertical posts, AB and CD .



Work out the distance AD .

[4 marks]

$$6.5 - 2.3 = 4.2$$



$$\sin 15 = \frac{4.2}{x}$$

$$x = \frac{4.2}{\sin 15}$$

$$= 48.18959563$$

Answer 48.2 m

- 16 During a game, players can win and lose counters.

At the start of the game

Rob, Tim and Zak share the counters in the ratio 5 : 6 : 7

At the end of the game

Rob, Tim and Zak share the **same number** of counters in the ratio 7 : 9 : 8

Show that Rob ends the game with more counters than he started with.

[3 marks]

| | | | | |
|-------|----|----|----|-----------------------|
| | R | T | Z | |
| start | 5 | 6 | 7 | = 18 $72 \div 18 = 4$ |
| | 20 | 24 | 28 | |
| | | | | assuming 72 counters |
| end | 7 | 9 | 8 | = 24 $72 \div 24 = 3$ |
| | 21 | 27 | 24 | |

if we assume 72 counters he starts with 20 and ends with 21

- 17

Factorise

$$3x^2 + 14x + 8 \quad \begin{array}{r} 3 \times 8 = 24 \\ 124 \\ 212 \end{array}$$

[2 marks]

$$3x^2 + 12x + 2x + 8$$

$$3x(x + 4) + 2(x + 4)$$

$$(3x + 2)(x + 4)$$

Answer $(3x + 2)(x + 4)$

18

Here is some information about the number of books read by a group of people in 2014

One of the frequencies is missing.

| Number of books | Frequency | Midpoint | |
|-----------------|-----------|----------|------|
| 0 – 4 | 16 | 2 | 32 |
| 5 – 9 | x | 7 | $7x$ |
| 10 – 14 | 20 | 12 | 240 |
| 15 – 19 | 10 | 17 | 170 |

$$46 + x$$

$$442 + 7x$$

Midpoints are used to work out an estimate for the mean number of books read.

The answer is 8.5

Work out the missing frequency.

[5 marks]

$$\frac{442 + 7x}{46 + x} = 8.5$$

$$442 + 7x = 8.5(46 + x)$$

$$442 + 7x = 391 + 8.5x$$

$$442 - 391 = 8.5x - 7x$$

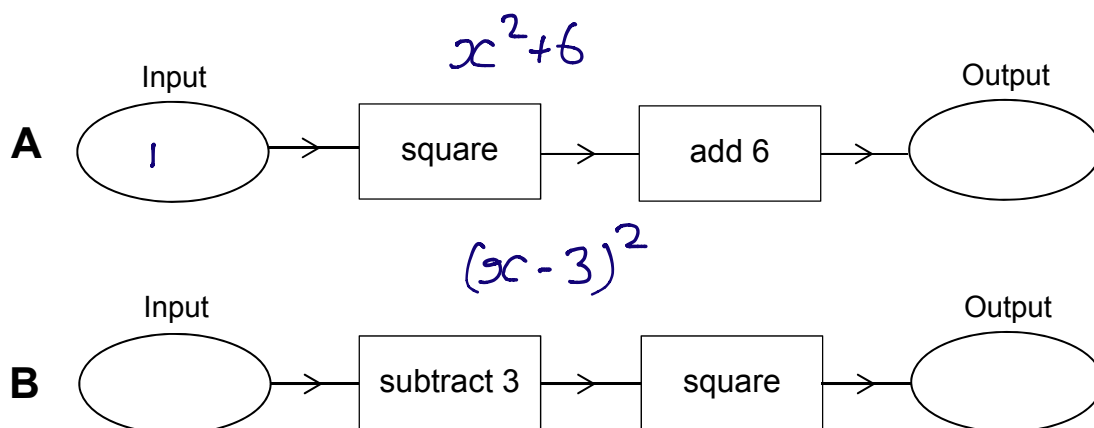
$$51 = 1.5x$$

$$x = 51 / 1.5 \quad x = 34$$

Answer

34

- 19 Here are two function machines, **A** and **B**.



Both machines have the same input.

Work out the range of input values for which

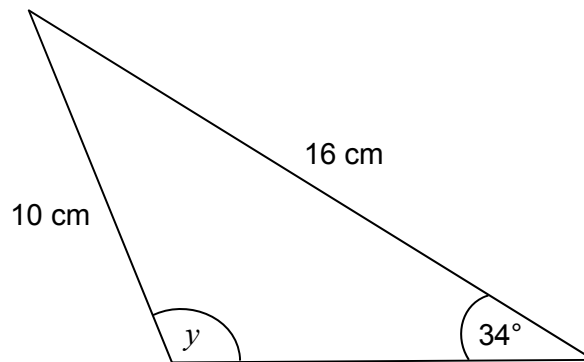
the output of **A** is **less** than the output of **B**.

[4 marks]

$$\begin{aligned}
 & x^2 + 6 & (x - 3)^2 \\
 & & x^2 - 6x + 9 \\
 & x^2 + 6 < x^2 - 6x + 9 \\
 & 6x < 3 \\
 & x < 0.5
 \end{aligned}$$

Answer $x < 0.5$

20

In the triangle, angle y is obtuse.Not drawn
accuratelyWork out the size of angle y .

[3 marks]

$$\frac{\sin 34}{10} = \frac{\sin y}{16}$$

$$\sin y = \frac{\sin 34}{10} \times 16 = 0.894 \dots$$

$$y = \sin^{-1}(0.894 \dots) = 63.47103073$$

obtuse angle!
 $180 - 63.47 \dots$

Answer 116.53 degrees

Turn over for the next question

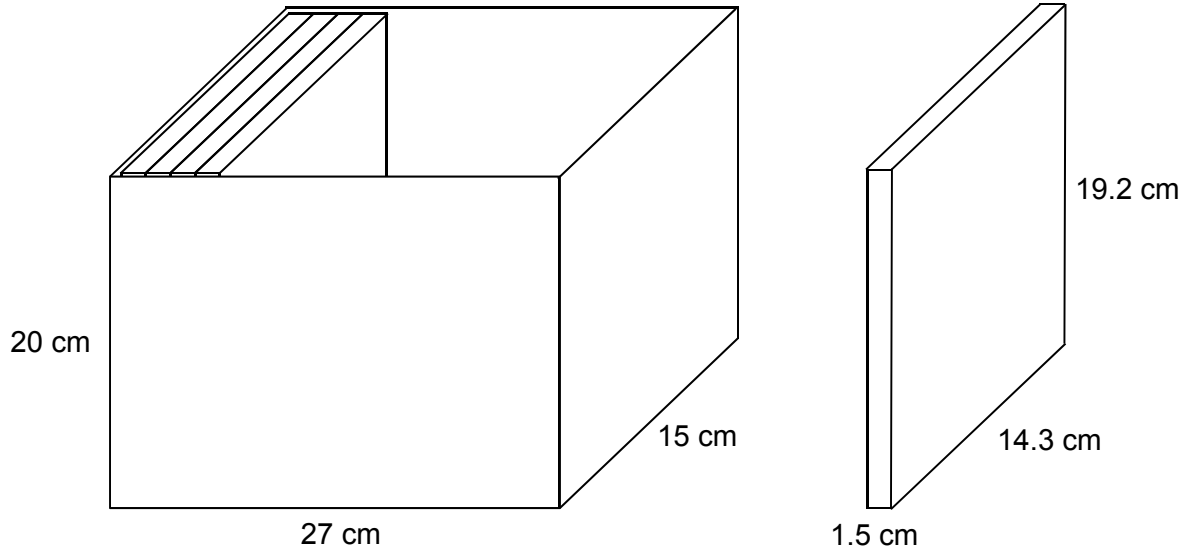
21

A box is a cuboid with dimensions 27 cm by 15 cm by 20 cm

These dimensions are to the nearest **centimetre**.

DVD cases are cuboids with dimensions 1.5 cm by 14.3 cm by 19.2 cm

These dimensions are to the nearest **millimetre**.



Show that 17 DVD cases, stacked as shown, will definitely fit in the box.

[4 marks]

$$1.5 \text{ cm} = 15 \text{ mm} \begin{cases} \nearrow 15.5 \text{ mm} \\ \searrow 14.5 \text{ mm} \end{cases} \quad \text{if biggest } 15.5 \times 17 = 263.5 \text{ mm}$$

$$27 \text{ cm} \begin{cases} \nearrow 27.5 \text{ cm} \\ \searrow 26.5 \text{ cm} \end{cases} \quad \begin{array}{l} \text{smallest width is } 26.5 \text{ cm and biggest} \\ \text{CD will be} = 26.35 \text{ cm} \\ \text{so will fit 17 CD's.} \end{array}$$

22

Bag X contains 9 blue balls and 18 red balls. 27

Bag Y contains 7 blue balls and 14 red balls. 21

Liz picks a ball at random from bag X.

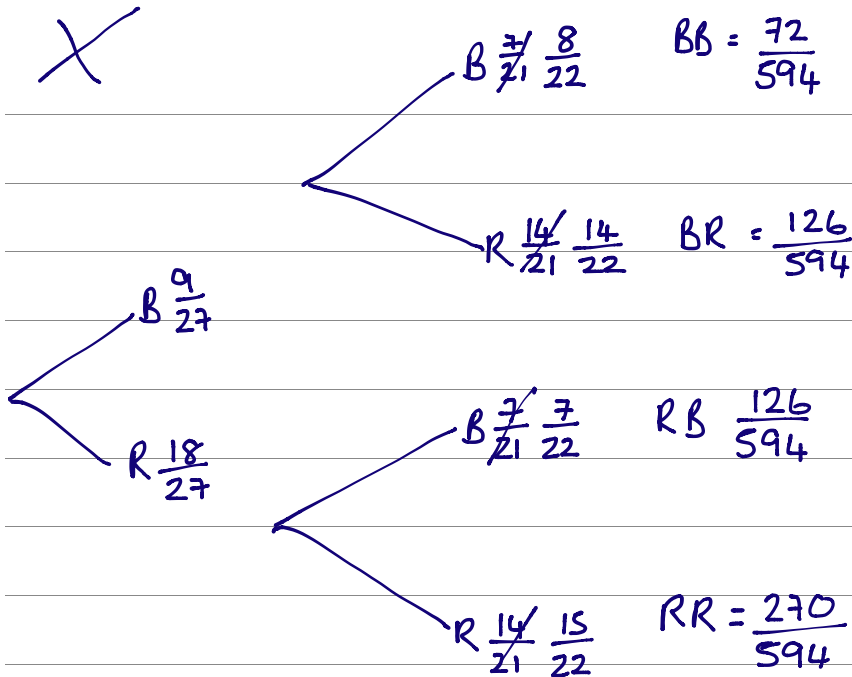
She puts the ball into bag Y.

Mike now picks a ball at random from bag Y.

Show that

$$P(\text{Liz picks a blue ball}) = P(\text{Mike picks a blue ball})$$

[4 marks]



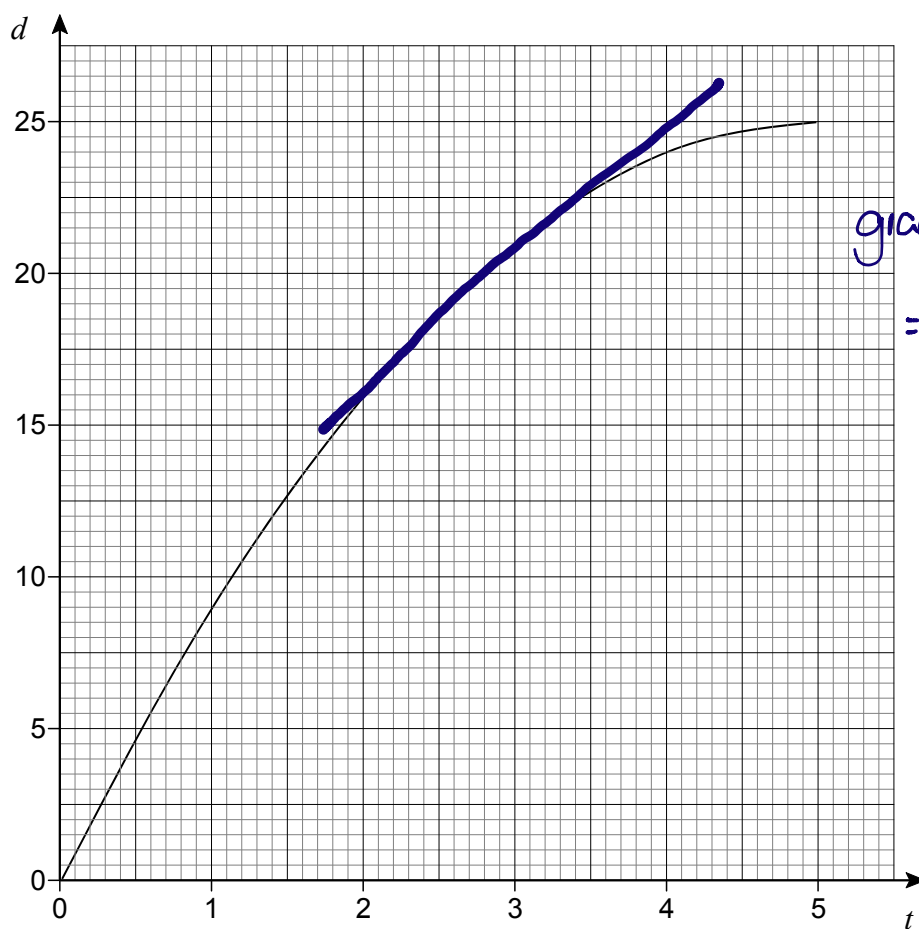
$$P(\text{Liz picking } B) = \frac{9}{27} = \frac{1}{3}$$

$$P(\text{Mike picking } B) = \frac{72}{594} + \frac{126}{594} = \frac{198}{594} = \frac{1}{3}$$

23

A container is filled with water in 5 seconds.

The graph shows the depth of water, d cm, at time t seconds.



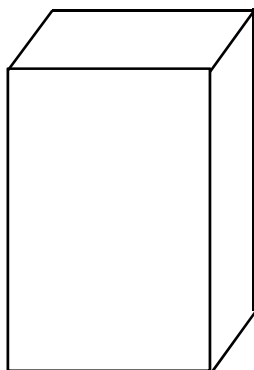
- 23 (a) The water flows into the container at a constant rate.

Which diagram represents the container?

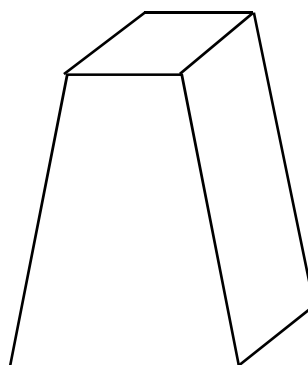
Circle the correct letter.

[1 mark]

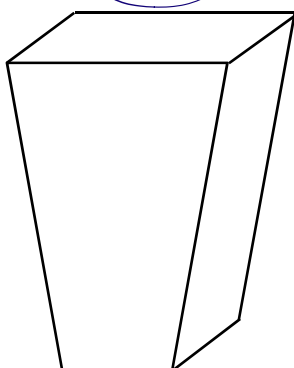
A



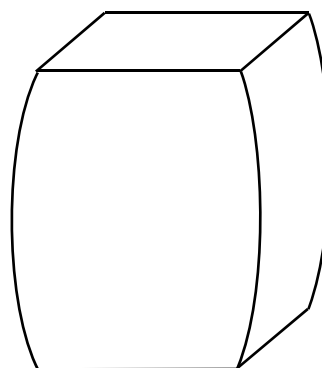
B



C



D



- 23 (b) Use the graph to estimate the rate at which the depth of water is increasing at 3 seconds. You **must** show your working.

[2 marks]

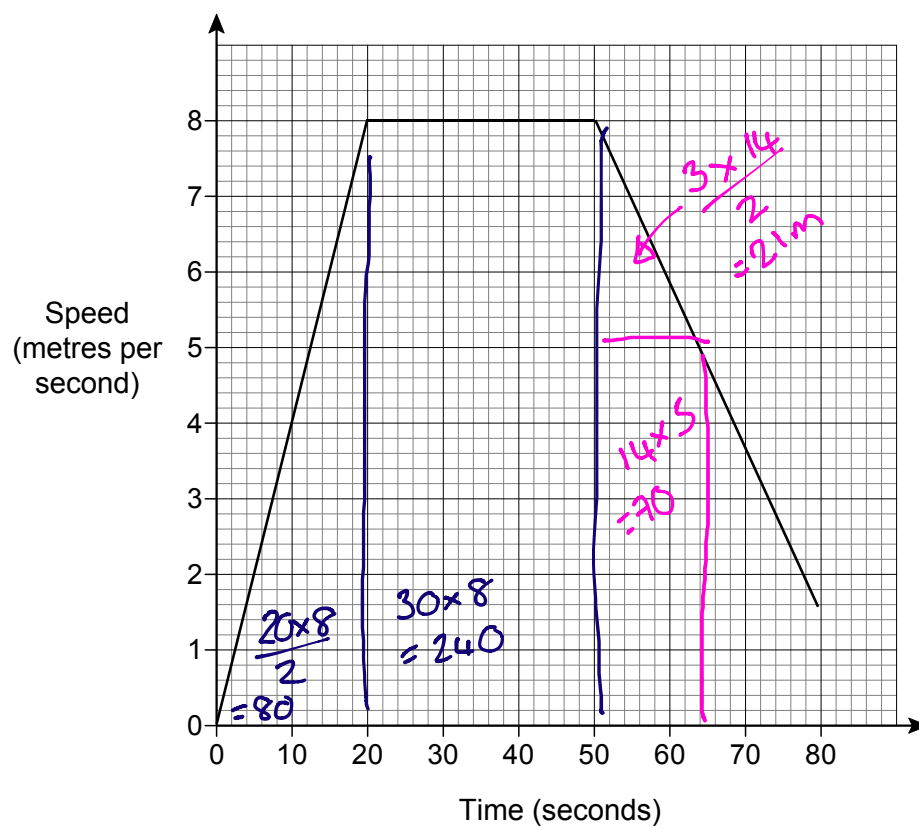
$$\text{gradient} = \frac{10}{2.6} = 3.846153$$

Answer 3.85 cm/s

24

Amina and Ben had a cycle race.

Here is Amina's speed-time graph from the start of the race.



24

The distance of the race was 400 metres.

Ben cycled the 400 metres in 64 seconds.

Who won the race?

You **must** show your working.

[4 marks]

in 64 seconds Amuna had done $80 + 240 + 70 + 21$
 $= 411\text{m}$

so Amuna won!

Answer _____

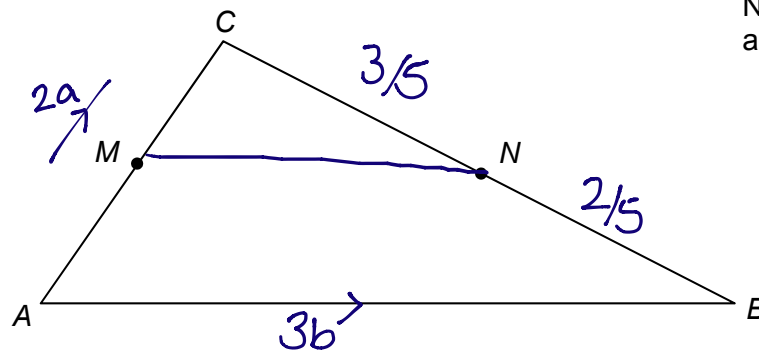
Turn over for the next question

25

In triangle ABC M is the midpoint of AC N is the point on BC where $BN : NC = 2 : 3$

$$\vec{AC} = 2\mathbf{a}$$

$$\vec{AB} = 3\mathbf{b}$$



Not drawn accurately

- 25 (a) Work out \vec{MN} in terms of \mathbf{a} and \mathbf{b} .
Give your answer in its simplest form.

[3 marks]

$$\vec{CB} = -2\mathbf{a} + 3\mathbf{b} = 3\mathbf{b} - 2\mathbf{a}$$

$$\vec{CN} = \frac{3}{5}(3\mathbf{b} - 2\mathbf{a})$$

$$\vec{MN} = \mathbf{a} + \frac{3}{5}(3\mathbf{b} - 2\mathbf{a})$$

$$= \mathbf{a} + \frac{9}{5}\mathbf{b} - \frac{6}{5}\mathbf{a} = \frac{1}{5}(9\mathbf{b} - \mathbf{a})$$

Answer $\frac{1}{5}(9\mathbf{b} - \mathbf{a})$

- 25 (b) Use your answer to part (a) to explain why MN is **not** parallel to AB .

[1 mark]

AB is not a multiple of MN so is not parallel

- 26** An approximate solution to an equation is found using this iterative process.

$$x_{n+1} = \frac{(x_n)^3 - 3}{8} \quad \text{and} \quad x_1 = -1$$

- 26 (a)** Work out the values of x_2 and x_3

[2 marks]

$$x_1 = -1$$

$$x_2 = \frac{(-1)^3 - 3}{8} = -\frac{1}{2}$$

$$x_3 = \frac{\left(-\frac{1}{2}\right)^3 - 3}{8} = -\frac{25}{64}$$

$$x_2 = -\frac{1}{2}$$

$$x_3 = -\frac{25}{64}$$

- 26 (b)** Work out the solution to 6 decimal places.

[1 mark]

$$x_4 = \frac{\left(-\frac{25}{64}\right)^3 - 3}{8} = -0.3824505806$$

$$x_5 = \frac{(-0.3824\dots)^3 - 3}{8} = -0.3819925565$$

$$x = -0.38199$$

27 The curve with equation $y = x^2 - 5x + 2$ is reflected in the x -axis.

Circle the equation of the reflected curve.

[1 mark]

$$y = x^2 - 5x - 2$$

$$y = -x^2 + 5x + 2$$

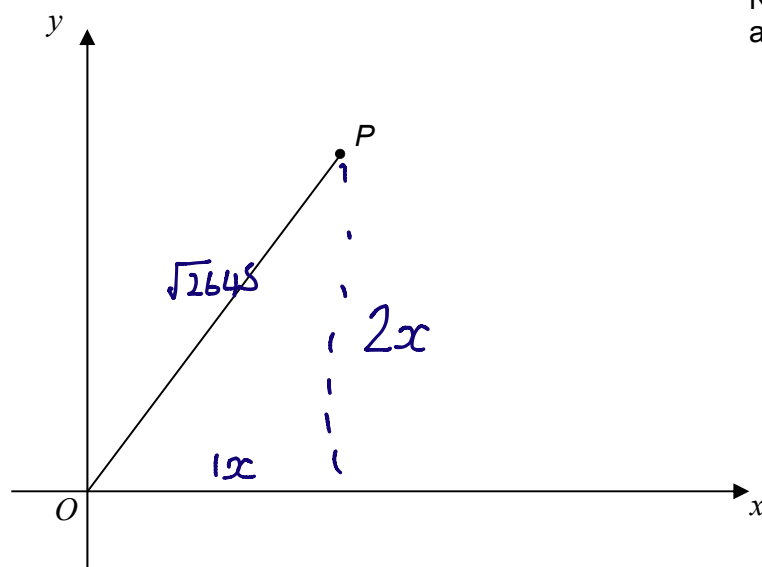
$$y = -x^2 + 5x - 2$$

$$y = x^2 + 5x + 2$$

28

The diagram shows a line joining O to P .

Not drawn
accurately



The gradient of the line is 2

The length of the line is $\sqrt{2645}$

Work out the coordinates of P .

$$(2x)^2 + x^2 = 2645$$

$$4x^2 + x^2 =$$

$$5x^2 = 2645$$

$$x^2 = 529$$

$$x = 23$$

[4 marks]

Answer (23 , 46)

END OF QUESTIONS