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H

GCSE (9–1) Mathematics
J560/04 Paper 4 (Higher Tier)
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

worked solutions

You may use:
• A scientific or graphical calculator
• Geometrical instruments
• Tracing paper

First name	*JustMaths*
Last name	

Centre number						Candidate number				

INSTRUCTIONS
• Use black ink. You may use an HB pencil for graphs and diagrams.
• Complete the boxes above with your name, centre number and candidate number.
• Answer **all** the questions.
• Read each question carefully before you start to write your answer.
• Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
• Write your answer to each question in the space provided.
• Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
• Do **not** write in the bar codes.

INFORMATION
• The total mark for this paper is **100**.
• The marks for each question are shown in brackets **[ ]**.
• Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
• This document consists of **20** pages.

J560/04

Turn over

Answer **all** the questions

1 18 rice cakes weigh a total of 130 g.
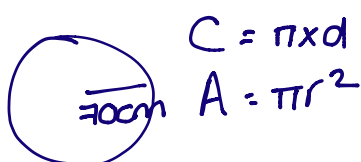There are 329 calories in 100 g of rice cakes.

How many calories are there in one rice cake?

÷18 (18rice cakes = 130g) ÷18
   1rice cake = 7·2g

329 calones = 100g) ÷100
÷100 3·29   = 1g
×7·2 (23·688 = 7·2g) ×7·2

........**23·7**........ calories **[3]**

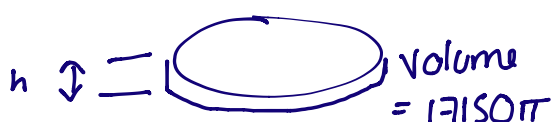**2** A circular table top has radius 70 cm.

(a) Calculate the area of the table top in cm^2, giving your answer as a multiple of π.

C = π×d   A = π×70^2
A = πr^2 = 4900π

70cm

(a) ......**4900π**...... cm^2 **[2]**

(b) The volume of the table top is 17 150π cm^3.

Calculate the thickness of the table top.

17150π = 4900π × h

h = $\frac{17150π}{4900π}$ = 3·5

Volume
= 17150π

h ⇕

(b) ......**3·5**...... cm **[2]**

3   The value of a car £V is given by

$$V = 20\,000 \times 0.9^t$$

where t is the age of the car in complete years.

(a)   Write down the value of V when $t = 0$.

$V = 20,000 \times 0.9^0$

$= 20,000 \times 1$

(a) £ 20,000 .... **[1]**

(b)   What is the value of V when $t = 3$?

$V = 20,000 \times 0.9^3$

(b) £ 14 580 .... **[2]**
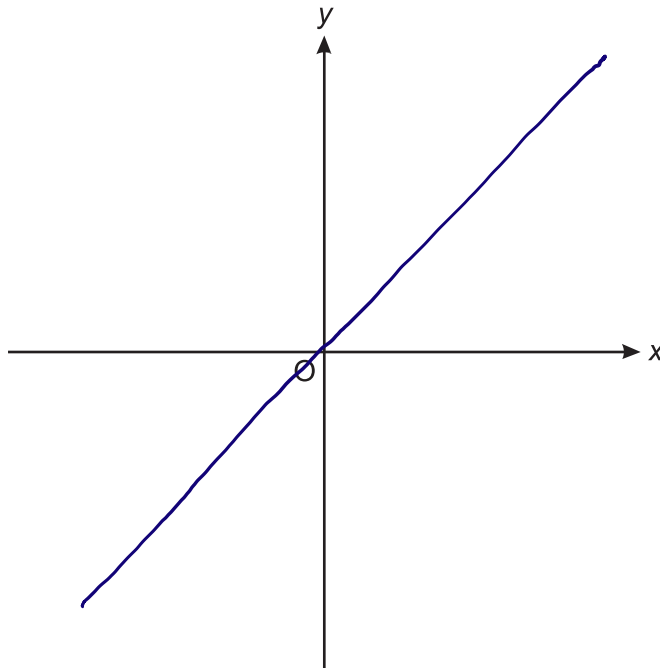
(c)   After how many complete years will the car's value drop below £10 000?

Year 3   value = 14 580

5   value = $20000 \times 0.9^5 = 11,809.80$

6   value = $20000 \times 0.9^6 = 10,628.82$
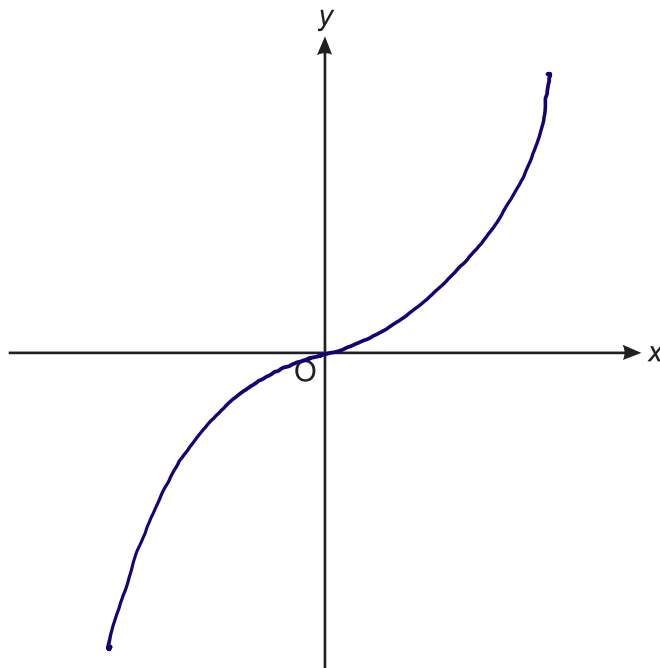
7   value = $20000 \times 0.9^7 = 9565.94$

(c) 7 years .... **[2]**

4 **(a)** **(i)** Sketch a graph on the axes below that shows that *y* is directly proportional to *x*.

[2]

(ii) Sketch a graph on the axes below that shows $y = x^3$.

[2]

$$x \quad -5 \quad 0 \quad 5$$
$$y^3 = -125 \quad 0 \quad 125$$

(b) It is possible to draw many rectangles that have area 24 cm².
Here are two of them.

2 cm

3 cm

12 cm

8 cm

Not to scale
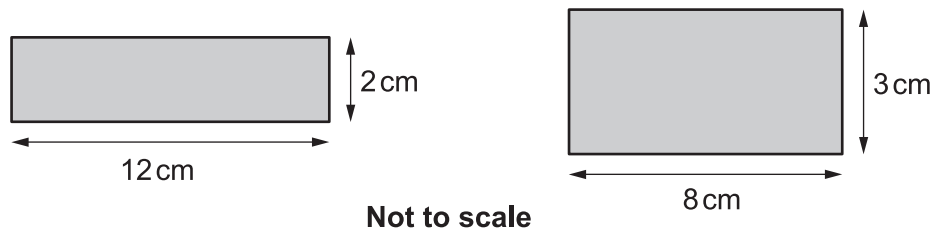
(i) Plot the dimensions of these two rectangles on the grid below. **[1]**

(ii) Complete the graph to show the relationship between length and width for rectangles with area 24 cm². **[3]**

Length (cm)

Width (cm)
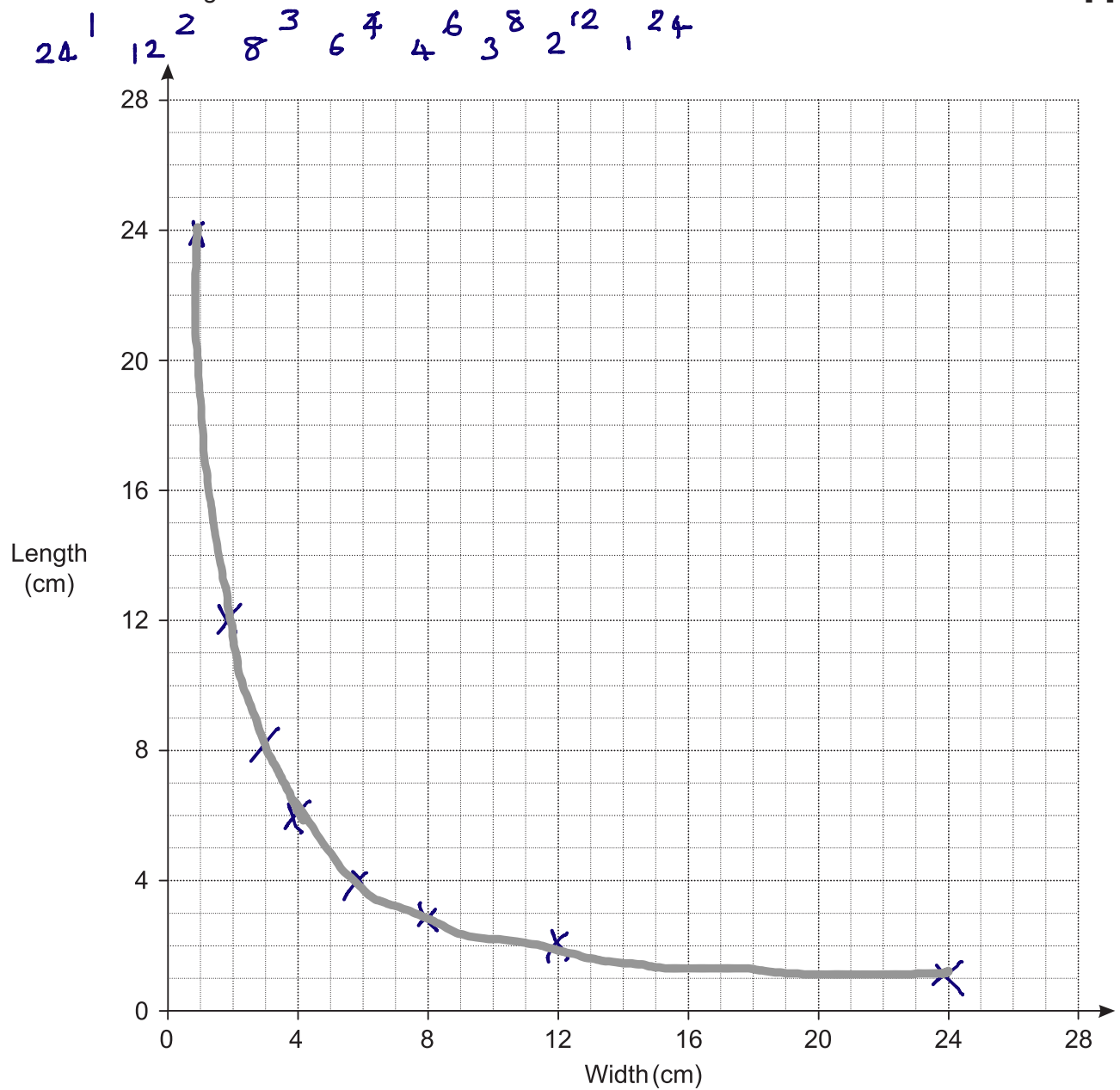
5 Kieran, Jermaine and Chris play football.

- Kieran has scored 8 more goals than Chris.
- Jermaine has scored 5 more goals than Kieran.
- Altogether they have scored 72 goals.

How many goals did they each score?

K = Kieran = Chris + 8
J = Jermaine = Kieran + 5
C = Chris =

$K + J + C = 72$

so $C = x$
$K = x + 8$
$J = x + 8 + 5$

$x + x + 8 + x + 13 = 72$
$3x + 21 = 72$
$3x = 72 - 21$
$3x = 51$
$x = 17$

$C = 17$
$K = 25$
$J = 30$

Kieran ....25..........

Jermaine ......30..........

Chris ......17..........

[5]

6 Peter makes a large amount of pink paint by mixing red and white paint in the ratio 2 : 3.

Red paint costs £80 per 10 litres.
White paint costs £5 per 10 litres.

Peter sells his pink paint in 10-litre tins for £60 per tin.

Calculate how much **profit** he makes for each tin he sells.

Red   white
2   3

£80   £5
10 litres   10 litres
= £8   = £0·50
per litre   per litre

To make 10   2 : 3   = 5 parts   10 ÷ 5 = 2
litres
            4 : 6
         litres   litres

    4 × £8 + 6 × 0·50
    £32   + £3 = £35

Profit £80 - £35 = £25

£ .... 25 .... **[5]**

7 Dan believes he knows what his brother Ethan is thinking.
He carries out an experiment to test this.

Dan and Ethan sit back-to-back.
Ethan rolls an ordinary fair dice.
Ethan then thinks about the number on the dice while Dan tries to predict this number.

(a) In 300 attempts, how many correct predictions would you expect Dan to make if he was just guessing?

equal chance for each side
of the dice = $\frac{1}{6}$

$\frac{1}{6} \times 300$

(a) ....50.... **[2]**

(b) The results of the first 15 attempts are shown in the table.

Ethan's number	2	6	5	3	2	1	5	1	3	4	4	6	1	6	5
Dan's prediction	2	4	3	1	2	6	1	6	4	3	2	6	5	2	3
Matching pair	✓				✓							✓			

first five.

Estimate the probability of getting a matching pair using the results of

(i) the first five attempts,

(b)(i) ....$\frac{2}{5}$.... **[1]**

(ii) all 15 attempts.

(ii) $\frac{3}{15} = \frac{1}{5}$ **[1]**

(c) Use answers from **(a)** and **(b)** to comment on Dan's belief that he knows what Ethan is thinking.

there is no evidence that Dan knows what Ethan is thinking because over 15 trials it tends to get closer to the theoretical probability of ⅙ **[2]**

8 **(a)** A function is represented by the following function machine.

Input — [× 2] — [+ 5] — Output

(i) A number is input into the machine.
The output is used as a new input.
The second output is 11.

Work out the number that was the **first input.**

$$\frac{3}{-1} \quad \div 2 \quad \frac{6}{-2} \quad -5 \leftarrow 11$$
$$\div 2 \quad -2 \quad -5 \leftarrow 3$$

(a)(i) ....-1.... **[2]**

(ii) A number is input into the machine.
The output given is the same number.

Work out the number.

$$x = 2x + 5$$
$$-5 = x$$

(ii) ....-5.... **[3]**

(b) Another function machine is shown below.

Input — [× 5] — [− 10] — Output

If the Input is 3, the Output is 5.
If the Input is 7, the Output is 25.

$$3 \times 2 = 6 - 1 = 5$$
$$7 \times 2 = 14 - 1 = 13 \ ✗$$

Use this information to fill in the two boxes.

$$\boxed{3} \times 3 = 9 - 4 = \boxed{5}$$ **[3]**
$$\boxed{7} \times 3 = 21 - 4 = 17 \ ✗$$

$$\boxed{3} \times 4 = 12 - 7 = \boxed{5}$$
$$\boxed{7} \times 4 = 28 - 7 = 21 \ ✗$$

$$\boxed{3} \times 5 = 15 - 10 = \boxed{5}$$
$$\boxed{7} \times 5 = 35 - 10 = \boxed{25}$$

9 **(a)** Anna estimates the height of a tree.
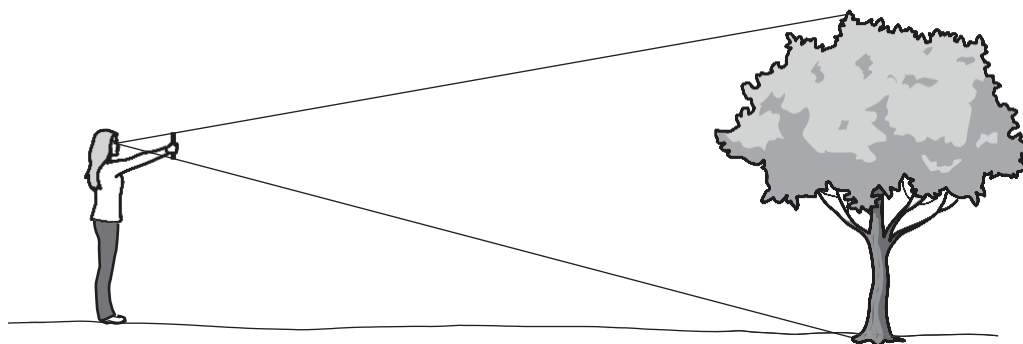
Anna holds a ruler vertically so the height of the tree is exactly covered by the ruler.
She is 20 metres from the tree.
The ruler is 30 cm long.
The horizontal distance from her eyes to the ruler is 60 cm.
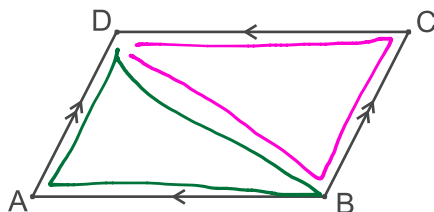
Calculate an estimate of the height of the tree.

scale factor
$= \dfrac{20}{0.6} = 33.\dot{3}$
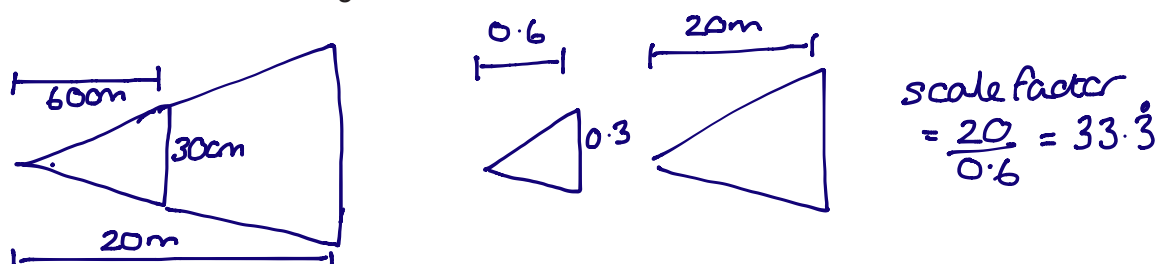
$0.3 \times 33.\dot{3}$

(a) ....10.......... m **[3]**

(b) Give two reasons why this method may not be suitable to estimate the height of a very tall building.

1 likely to be inaccurate.

2 She would need to be a long way from the building

[2]

10   ABCD is a parallelogram.

Prove that triangle ABD is congruent to triangle CDB.   [3]

Side BD is a common side so is equal   (s)

AB = DC as its a parallelogram   (S)

$\widehat{ABD}$ = $\widehat{CDB}$ alternate angles are equal (A)

ABD is congruent to CDB due to SAS

11 (a) Give **one** reason why 0 is an even number.

- 1 is odd, 1 is odd and 0 is between there two numbers, and the numbers go odd, even, odd, even etc [1]

(b) The lengths of the sides of a **right-angled** triangle are all integers.

Prove that if the lengths of the two shortest sides are even, then the length of the third side must also be even.
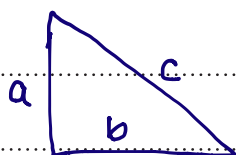
$c^2 = a^2 + b^2$

Let $a = 2x$ (always an even number)

$b = 2y$ (always an even number)

$c^2 = (2x)^2 + (2y)^2 = 4x^2 + 4y^2$

$c^2 = 4(x^2 + y^2)$ ← will always be even [3]

12 (a) Without using a calculator, show that $\sqrt{20} = 2\sqrt{5}$. [2]
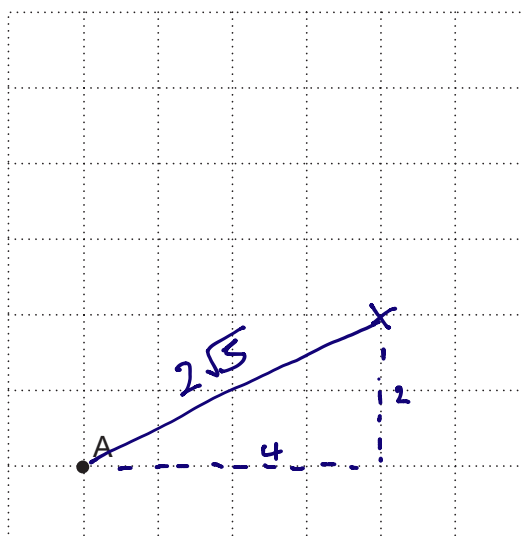
$\sqrt{20} = \sqrt{4} \times \sqrt{5}$

$2\sqrt{5}$

(b) The point A is shown on the unit grid below.
The point B is $2\sqrt{5}$ units from A and lies on the intersection of two grid lines.

Mark **one** possible position for B.

$c^2 = a^2 + b^2$

$20 = 2^2 + 4^2$

$4 + 16 = 20$

$\sqrt{20} = 2\sqrt{5}$

[3]

13 The volume of Earth is 1.08×10^{12} km^3.
The volume of Jupiter is 1.43×10^{15} km^3.

How many times larger is the radius of Jupiter than the radius of Earth?
Assume that Jupiter and Earth are both spheres.

[The volume v of a sphere with radius r is $v = \frac{4}{3}\pi r^3$.]

$$\frac{3v}{4} = \pi r^3$$

$$r = \sqrt[3]{\frac{3v}{4\pi}}$$

Earth...

$$r = \sqrt[3]{\frac{3 \times 1.08 \times 10^{12}}{4\pi}} = 6364.706508$$

Jupiter

$$r = \sqrt[3]{\frac{3 \times 1.43 \times 10^{15}}{4\pi}} = 69890.12374$$

$$\frac{69890}{6365} = 10.980\ldots$$

11 .... **[4]**

14 The table shows the marks gained by 150 students taking an examination.

Mark (m)	0<m≤10	10<m≤20	20<m≤30	30<m≤40	40<m≤50	50<m≤60	60<m≤70	70<m≤80
Frequency	9	14	26	27	25	22	17	10

(a) (i) Construct a cumulative frequency table.

Mark (m)	m ≤ 10	m ≤ 20	m ≤ 30	m ≤ 40	m ≤ 50	m ≤ 60	m ≤ 70	m ≤ 80
Cumulative Frequency	9	23	49	76	101	123	140	150

[2]

(ii) Draw the cumulative frequency graph on the grid below.
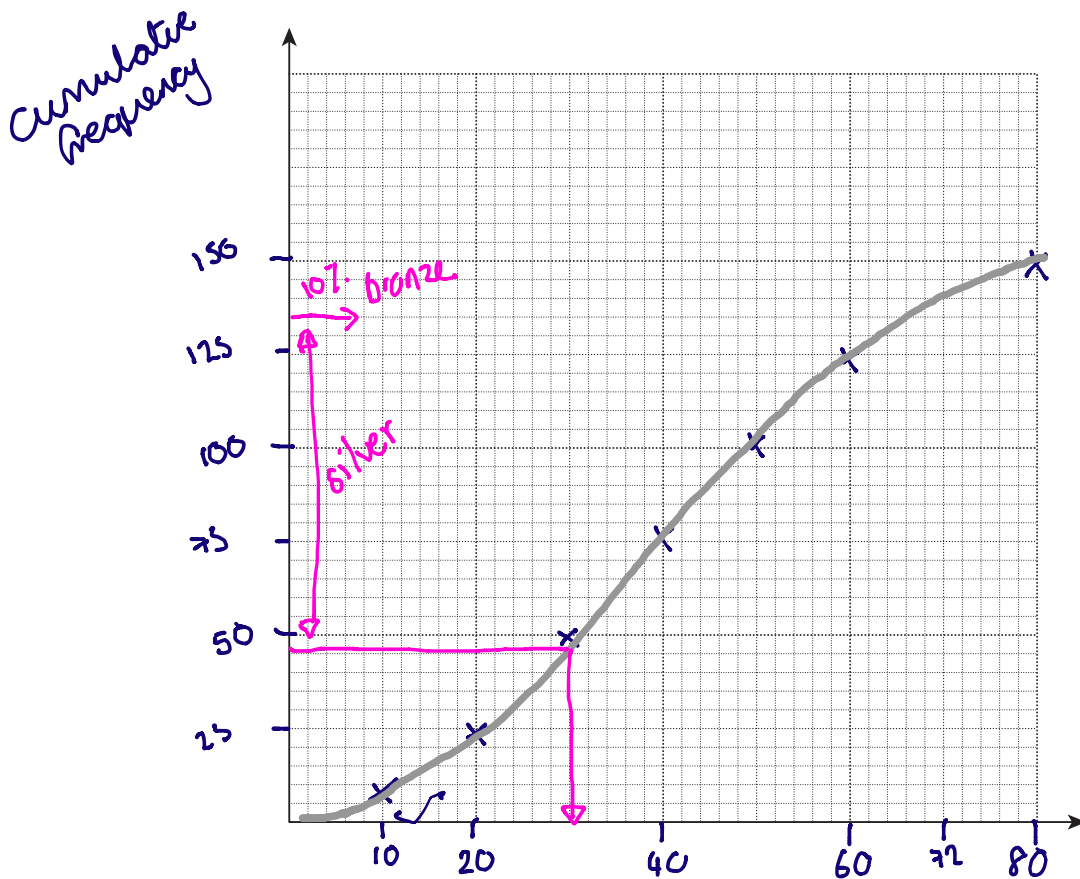
[4]

(b) Students are to be awarded Gold, Silver, Bronze or Fail.
The students' teacher wishes to award the top 10% of students Gold, the next 60% Silver and the next 20% Bronze.

Use your graph to estimate the lowest mark that Silver will be awarded for.

(b) ...._32_........ **[3]**

(c) Explain why the teacher's method will not necessarily award Gold to exactly 10% of the students.

the boundaries are based on grouped data

.... **[1]**

15 At a constant temperature, the volume of a gas V is inversely proportional to its pressure p.

By what percentage will the pressure of a gas change if its volume increases by 25%?
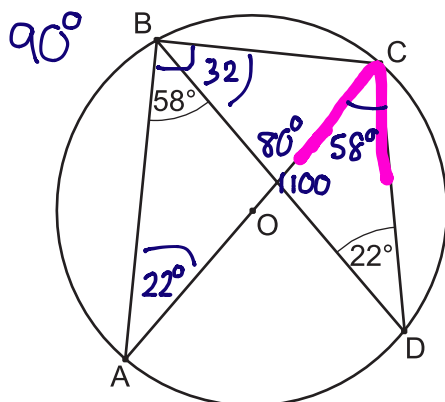
$$V = \frac{k}{p} \quad \leftarrow \text{constant}$$

$$V \times p \text{ (initial)} = V \times p \text{ (after)}$$

$$1 \times 1 \quad = 1.25 \times p$$

$$\frac{1}{1.25} = 0.8$$

20% decrease. .... % **[4]**

16 A, B, C and D are points on the circumference of a circle, centre O.
AC is a diameter of the circle.
Angle ABD = 58°.
Angle CDB = 22°.

90°

B
32)
58°
80° 58°
100
O
22°
22°
D
A
C

Not to scale

Work out the sizes of angle ACD and ACB, giving reasons for your answers.

(a) Angle ACD = ...58............°

angles subtended on same arc of a circle are equal

.... **[2]**

(b) Angle ACB = ...68............°

$\hat{ABC}$ = 90 angle in a semi-circle are 90°
$\hat{DBC}$ = 90·58 = 32°
$\hat{COD}$ = 100° (angles in a triangle = 180°) **[3]**
$\hat{BOC}$ = 80° angles on a straight line
$\hat{ACB}$ = 80+32 = 112
180 −112 = 68°

17 A restaurant menu has 8 starters, 12 mains and 6 desserts.
A customer can choose from the following meals

- a starter and a main,
- a main and a dessert,
- a starter, a main and a dessert.

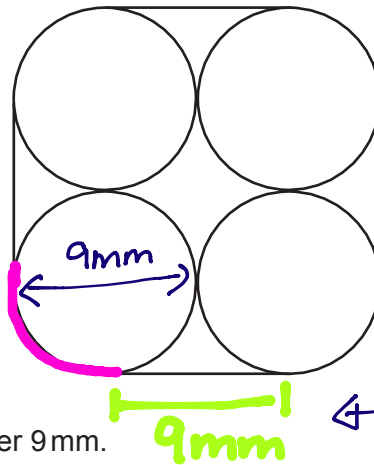Show that there are 744 different ways of choosing a meal at this restaurant. **[3]**

Starter and main combos $= 8 \times 12 = 96$

Main + desert combos $= 12 \times 6 = 72$

3 courses $= 8 \times 12 \times 6 = 576$

$96 + 72 + 576$
$= 744$

18 Four pencils are held together with a band.
The figure below shows the bottom end of the pencils and the band.

9mm

9mm ← ×4 = 36mm

Each of the pencils has diameter 9 mm.

Find the length of the band in this position.

Four arcs = 1 circle

$$C = \pi \times 9 = 9\pi$$

$$\text{Length} = 9\pi + 36$$
$$= 64.27433388$$

64.3 .... mm **[4]**

19 A sequence is defined by the term-to-term rule $u_{n+1} = u_n^2 - 8u_n + 17$.

(a) Given that $u_1 = 4$, find u_2 and u_3.

$U_2 = 4^2 - 8 \times 4 + 17$
$\quad = 16 - 32 + 17 = \underline{\underline{1}}$

$U_3 = 1^2 - 8 \times 1 + 17$
$\quad = 1 - 8 + 17 = \underline{\underline{10}}$

(a) ....1........ ....10........ [2]

(b) Given instead that $u_1 = 2$, find u_2, u_3 and u_{100}.

$U_2 = 2^2 - 8 \times 2 + 17$
$\quad = 4 - 16 + 17 = 5$
$U_3 = 5^2 - 8 \times 5 + 17$
$\quad = 25 - 40 + 17 = 2$
$U_4 = 2^2 - 8 \times 2 + 17$
$\quad = 4 - 16 + 17$

(b) ......5...... ......2...... ......5...... [3]

20 **(a)** Express as a single fraction.

$$\frac{m+1}{n+1} - \frac{m}{n}$$

Simplify your answer.

$$\frac{n(m+1) - m(n+1)}{n(n+1)} = \frac{mn+n-mn-m}{n(n+1)}$$

$$\frac{n-m}{n(n+1)}$$

(a) .... **[2]**

(b) Using your answer to part **(a)**, prove that if m and n are positive integers and $m < n$, then

$$\frac{m+1}{n+1} - \frac{m}{n} > 0.$$ **[2]**

$$\Rightarrow \frac{n-m}{n(n+1)} > 0$$

if $m < n$

$n - m > 0$

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