

GCSE (9–1) Mathematics J560/05 Paper 5 (Higher Tier) Sample Question Paper

H

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

Worked Solutions (M)



You may use:

- Geometrical instruments
- Tracing paper

Do not use:

- A calculator



First name	Just Maths				
Last name					
Centre number					
Candidate number					

INSTRUCTIONS

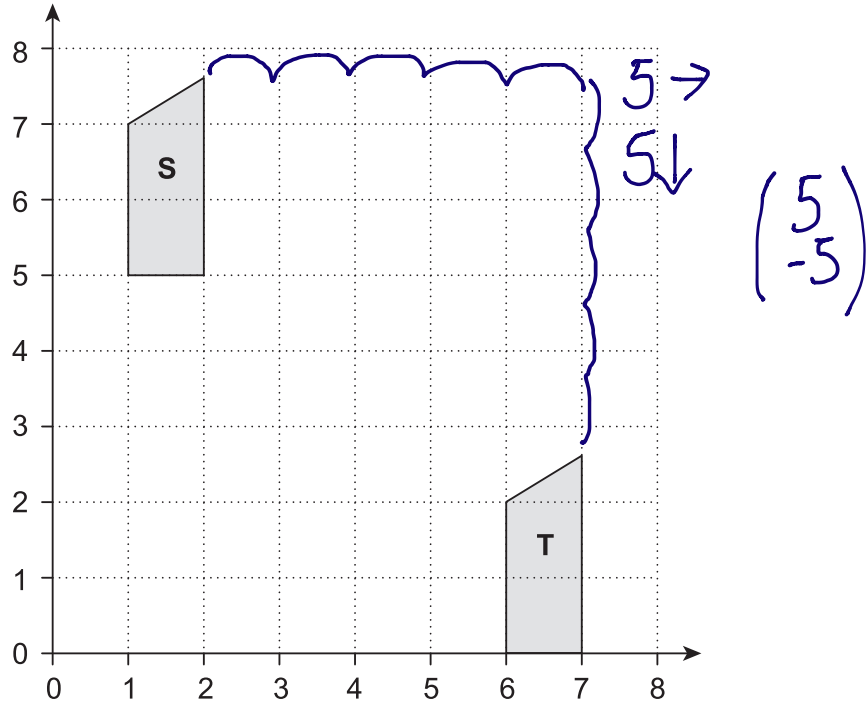
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document consists of **20** pages.

Answer all the questions

1 (a) Here is a coordinate grid.

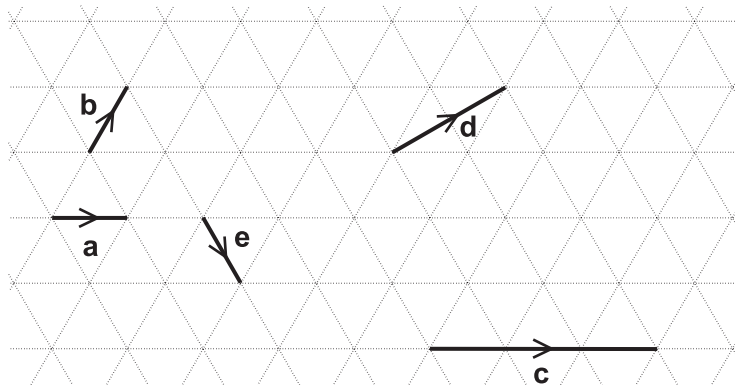


Shape S is translated to Shape T using vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the values of p and q .

(a) $p = \dots 5 \dots$
 $q = \dots -5 \dots$ [2]

(b) Vectors a , b , c , d and e are drawn on an isometric grid.



Write each of the vectors c , d and e in terms of a and/or b .

$c = \dots 3a \dots$
 $d = \dots a + b \dots$
 $e = \dots a - b \dots$

[3]

- 2 Sam and two friends put letters in envelopes on Monday.
The three of them take two hours to put 600 letters in envelopes.

$$\begin{aligned} 600 &= 2 \text{ hours} \\ 300 &= 1 \text{ hour} \\ 3 \text{ people} &= 300 \text{ in 1 hour} \\ 1 \text{ person} &= 100 \end{aligned}$$

- (a) On Tuesday Sam has three friends helping.

Working at the same rate, how many letters should the **four** of them be able to put in envelopes in two hours?

$$\begin{aligned} 1 &= 100 \\ 4 &= 400 \text{ an hour} \\ &\times 2 \end{aligned}$$

(a) 800 [2]

- (b) Working at the same rate, how much longer would it take **four** people to put 1000 letters in envelopes than it would take **five** people?

$$1000 \div 100 = 10 \text{ hours}$$

$$4 \text{ people} = 2\frac{1}{2} \text{ hours}$$

$$5 \text{ people} = 2 \text{ hours}$$

(b) 30 mins [4]

- (c) Sam says

It took two hours for three people to put 600 letters in envelopes.

If I assume they work all day, then in one day three people will put 7200 letters in envelopes because $600 \times 12 = 7200$.

Why is Sam's assumption not reasonable?

What effect has Sam's assumption had on her answer?

Sam is assuming they'll work for 12 hours.. it will take longer [2]

- 3 Abi, Ben and Carl each drop a number of identical drawing pins, and count how many land with the pin upwards. The table shows some of their results.

	Number of pins dropped	Number landing 'pin up'	
Abi	10	4	$\frac{4}{10}$
Ben	30	9	$\frac{9}{30}$
Carl	100	35	$\frac{35}{100}$

- (a) Abi says

As a drawing pin can only land with its pin up or with its pin down, the probability of a drawing pin landing 'pin up' is $\frac{1}{2}$.

Criticise her statement.

The outcomes are unlikely to be unbiased/equally likely to happen [1]

- (b) Carl's results give the best estimate of the probability of a drawing pin landing 'pin up'. Explain why.

he had more in his sample [1]

- (c) Two pins are dropped.

Estimate the probability that both pins land 'pin up'.

$$\frac{48}{150} \times \frac{48}{150} = \frac{64}{625}$$

140 → 150

(c) [2]

4 John is going to make chocolate squares to sell.

There are just three ingredients, chocolate, peanut butter and crisped rice, mixed in the ratio 4 : 2 : 3 respectively.

(a) How much of each ingredient will he need to make 900g of mixture?

$$\begin{array}{l} C : P : R \\ 4 : 2 : 3 = 9 \text{ parts} \\ \underbrace{\hspace{10em}} \\ 900\text{g} \div 9 = 100\text{g} \end{array}$$

$$4 \times 100 \quad 2 \times 100 \quad 3 \times 100$$

(a) chocolate 400 g
 peanut butter 200 g
 crisped rice 300 g

[2]

(b) A bar of chocolate weighs 200g and costs £2.50.
 A jar of peanut butter contains 250g and costs £1.70.
 A packet of crisped rice contains 300g and costs £2.00.

John makes 4.5 kg of mixture, from which he can cut 100 chocolate squares.
 He charges 60p for each square and sells all 100 squares.

$$0.60 \times 100 \\ \pounds 60.00$$

How much **profit** does he make?

$$\begin{array}{l} C \quad P \quad R \\ 4 : 2 : 3 \\ 4500 \div 9 = 500\text{g} \end{array}$$

$$1500 \quad 750 \quad 1500$$

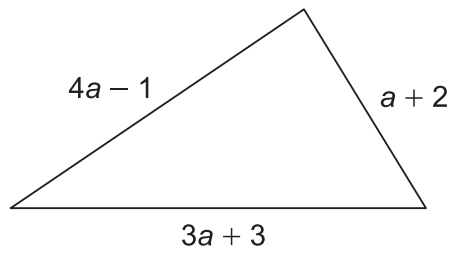
requires:-

2000	1000	1500
10 jars	4 jars	5 jars
$\times 2.50$	$\times 1.70$	$\times 2.00$
£25	£6.80	£10
$\underbrace{\hspace{10em}}$		
£41.80		

$$60 - 41.80$$

(b) £ 18.20 [5]

- 5 The perimeter of the triangle is the same length as the perimeter of the square.



Find an expression for the length of one side of the square in terms of a .
Give your answer in its simplest form.

$$\begin{aligned} \text{Perimeter of triangle} \\ = 8a + 4 \end{aligned}$$

$$\text{One side of square} = \frac{8a + 4}{4}$$

$$\underline{2a + 1} \dots \dots \dots [4]$$

- 6 A bag contains only red and blue marbles.

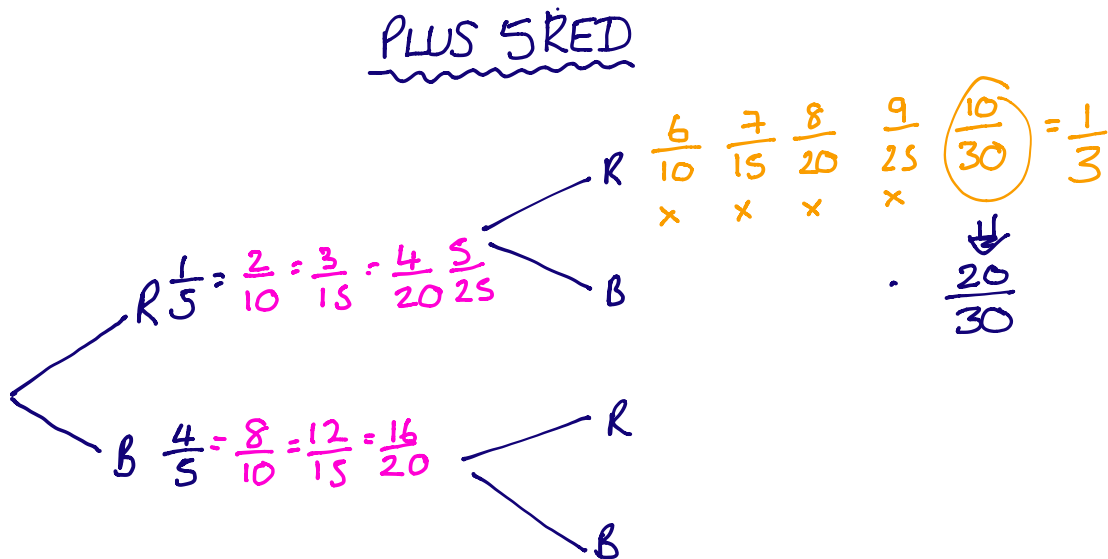
Yasmine takes one marble at random from the bag.

The probability that she takes a red marble is $\frac{1}{5}$.

Yasmine returns the marble to the bag and adds five more red marbles to the bag.

The probability that she takes one red marble at random is now $\frac{1}{3}$.

How many marbles of each colour were originally in the bag?



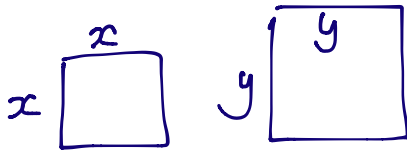
2nd
 10 red 20 Blue .
 less 5 red
 5 red 20 blue

..... 5 red marbles
 20 blue marbles

[3]

- 7 The lengths of the sides of two squares are integers, when measured in cm. The difference between the areas of the two squares is 36 cm^2 .

Find the lengths of the sides of the two squares.



$$10^2 - 8^2 = 100 - 64 = 36$$

$$\begin{array}{r} 10 \\ \hline 8 \\ \hline \end{array} \text{ cm}$$

[3]

- 8 Safety rules on a campsite require Sarah to set up her barbecue at least 4 m from her tent. She decides to measure this distance using her stride length. Sarah knows that her stride length is 0.8 m, rounded to the nearest 0.1 m.

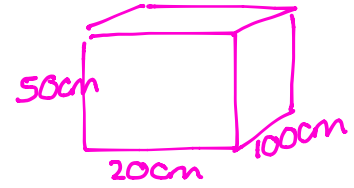
Find the minimum number of strides Sarah will need to take to **guarantee** that her barbecue is a safe distance from her tent.

$$0.8 \text{ m} \begin{cases} \nearrow \text{LB} = 0.75 \\ \rightarrow 0.1 \\ \searrow \text{UB} = 0.85 \end{cases}$$

$$4 \text{ m} \div 0.75 = 5.\dot{3}$$

6 strides [3]

- 9 A sculptor needs to lift a piece of marble.
 It is a cuboid with dimensions 1 m by 0.5 m by 0.2 m.
 Marble has a density of 2.7 g/cm^3 .
 The sculptor's lifting gear can lift a maximum load of 300 kg.



Can the lifting gear be used to lift the marble?
 Justify your decision.

$$\text{Volume} = 100,000 \text{ cm}^3$$

$$D = \frac{m}{V}$$

$$m = D \times V$$

$$\begin{aligned} m_{\text{max}} &= 2.7 \times 100,000 \\ &= 270,000 \\ &= 270 \text{ kg} \end{aligned}$$

Yes the lifting gear can be used

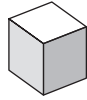
..... [4]

10 Here is a picture of three towers.
Not all the cubes can be seen in the towers.

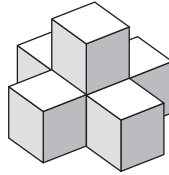
n = 1
palatun = 1

2
6

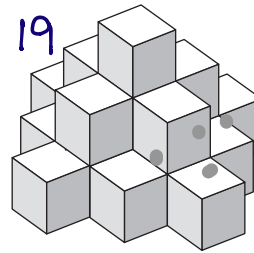
3
19



Tower 1



Tower 2



Tower 3

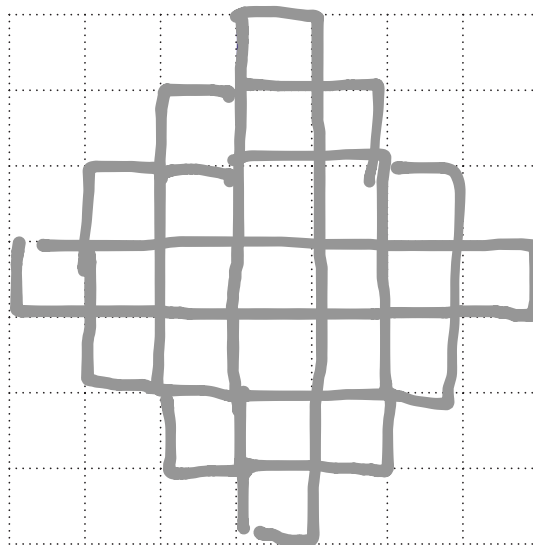
Edith uses 1 cube to build tower 1.

Edith uses 6 cubes to build tower 2. There are 5 cubes on the bottom layer.

(a) Write down the total number of cubes in tower 3.

(a) 19 [1]

(b) Draw a plan view of the arrangement of cubes Edith will use for the bottom layer of tower 4.



[1]

(c) Continue this sequence to show the number of cubes used for the bottom layer of each tower.

Tower 1	Tower 2	Tower 3	Tower 4
1	5	13	25

[2]

(d) Find an expression for the number of cubes used in the bottom layer of tower n .

	1	2	3	4
	1	5	13	25
		4	8	12
		4	4	
$2n^2$	2	8	18	32
$-2n$	-2	-4	-6	-8
$+1$	1	5	13	25

$4 \div 2$

(d) $2n^2 - 2n + 1$ [4]

11 A toy car is placed on the floor of a sports hall.

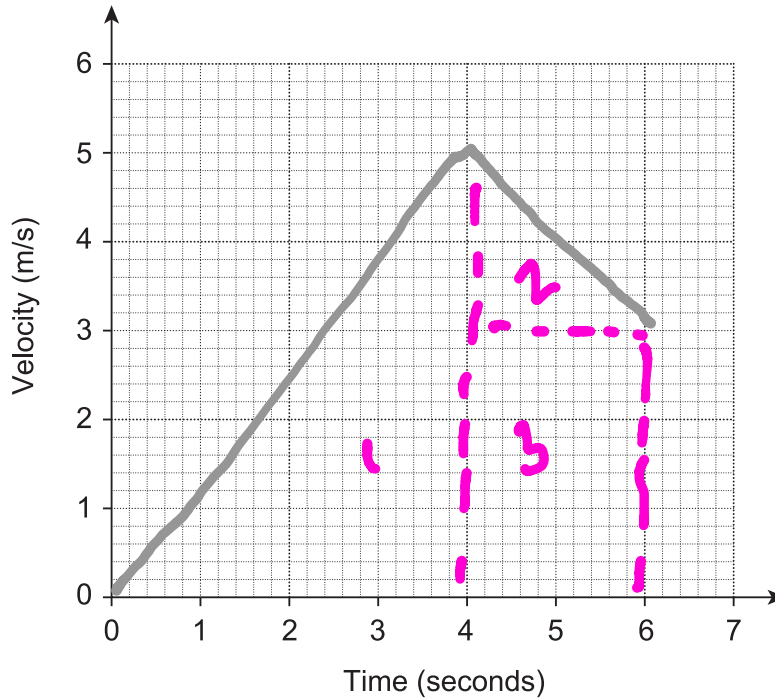
It moves in a straight line starting from rest.

It travels with constant acceleration for 4 seconds reaching a velocity of 5 m/s.

It then slows down with constant deceleration of 1 m/s^2 for 2 seconds.

It then hits a wall and stops.

(a) Draw a velocity-time graph for the toy car.



$$a = \frac{v - u}{t} \quad -1 = \frac{v - 4}{2}$$

$$-2 + 4 = v \\ v = 2$$

[3]

(b) Work out the total distance travelled by the toy car.

$$\textcircled{1} = \frac{5 \times 4}{2} = 10$$

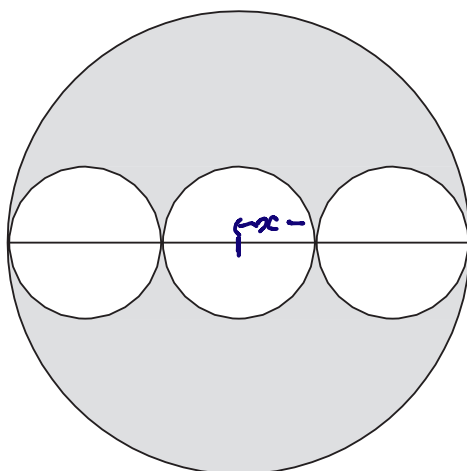
$$\textcircled{2} = \frac{2 \times 2}{2} = 2$$

$$\textcircled{3} = 2 \times 3 = 6$$

$$10 + 2 + 6$$

(b) 18 m [3]

- 12 Three identical small circles are drawn inside one large circle, as shown in the diagram. The centres of the small circles lie on the diameter of the large circle.



Find the fraction of the large circle that is shaded.

Let radius of small circle = x

$$\text{area of 1 circle} = \pi x^2$$

$$3 \text{ circles} = 3\pi x^2$$

Large circle radius = $3x$

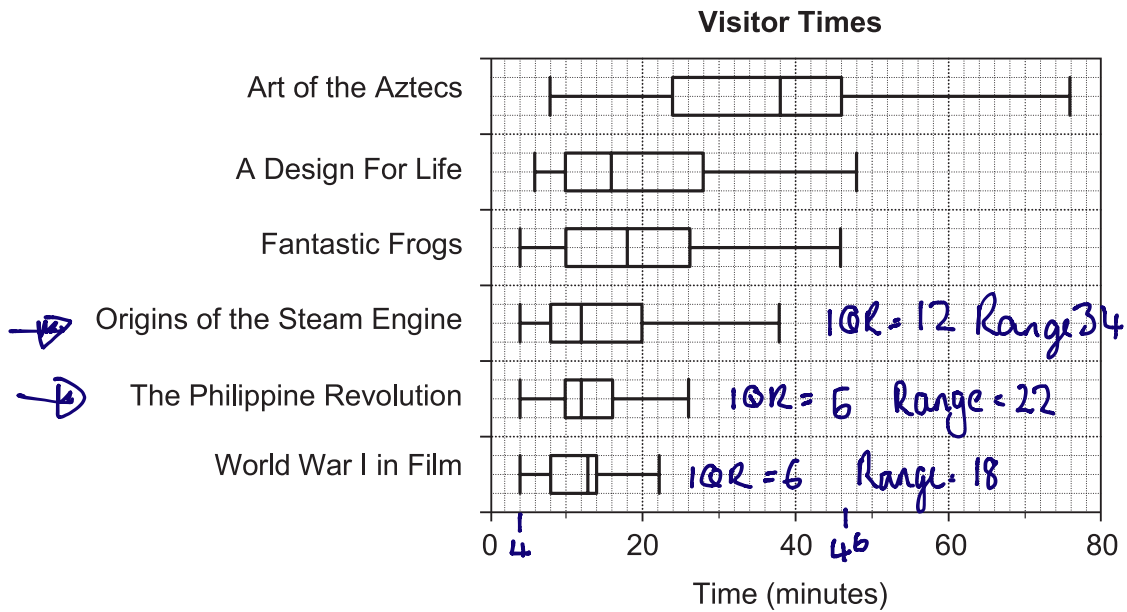
$$\text{Area} = \pi \times (3x)^2 = 9\pi x^2$$

$$\frac{3\pi x^2}{9\pi x^2} = \frac{1}{3} \text{ shaded}$$

$$\frac{2}{3} \text{ unshaded}$$

..... [3]

- 13 One day a museum monitored the time spent by visitors at six exhibitions. The visitor times are summarised in the box plots below.



- (a) Work out the **range** in visitor times at the **Fantastic Frogs** exhibition.

$$46 - 4$$

(a) 42 [2]

- (b) At which exhibition were visitor times the most consistent?
Give a reason for your answer.

World War I in film as it has the smallest IQR and range.

..... [2]

- (c) Give one similarity and one difference between the **distributions** of the visitor times for **Origins of the Steam Engine** and **The Philippine Revolution**.

Similarity They have the same median

Difference The Philippine revolution is more consistent as it has a smaller range of visitor times

[2]

- (d) Is it possible to work out from the box plots which exhibition had the most visitors? Justify your answer.

no as box plots show proportions and not frequencies

[2]

- 14 Show that line $3y = 4x - 14$ is perpendicular to line $4y = -3x + 48$.

[4]

$$l_1 \Rightarrow 3y = 4x - 14$$

$$y = \frac{4x}{3} - \frac{14}{3}$$

$$l_2 \Rightarrow 4y = -3x + 48$$

$$y = \frac{-3x}{4} + \frac{48}{4}$$

$$= \frac{-3x}{4} + 12$$

$$\text{gradient of } l_1 = \frac{4}{3}$$

$$\text{gradient of } l_2 = \frac{-3}{4}$$

$$\frac{4}{3} \times \frac{-3}{4} = -1$$

\therefore the lines are perpendicular

- 15 (a) Write this list of numbers in order, smallest first.

$$\sqrt{35}, \frac{20}{3}, 2.5^2, 6.83$$

$$6.\dot{6} \quad 6.25 \quad 6.83$$

$$3 \overline{) 20.0000}$$

$$\begin{array}{r} 6 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \end{array}$$

$$\sqrt{36} = 6$$

(a) $\sqrt{35}$ 2.5^2 $\frac{20}{3}$ 6.83 [2]
smallest

- (b) Write $(1 + \sqrt{3})^2$ in the form $a + b\sqrt{3}$.

$$(1 + \sqrt{3})(1 + \sqrt{3})$$

$$1 + \sqrt{3} + \sqrt{3} + 3$$

$$4 + 2\sqrt{3}$$

(b) $4 + 2\sqrt{3}$ [3]

- 16 Bethany says that $(2x)^2$ is always greater than or equal to $2x$.

Decide whether she is correct or not.

Show your working to justify your decision.

[3]

when $x = 1$	$(2x)^2$	$2x$	
	4	2	$(2x)^2 > 2x$

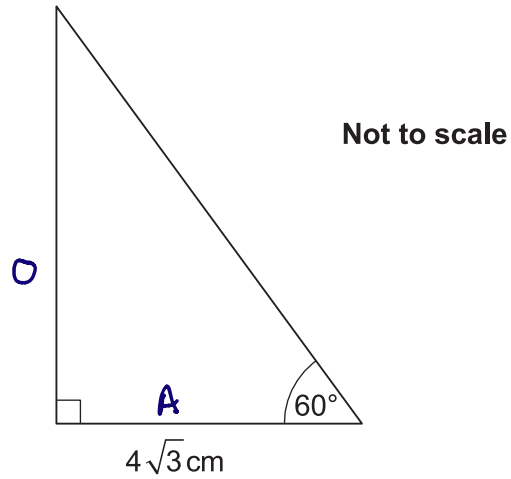
$x = 0.5$	1	1	$(2x)^2 = 2x$
$x = 0$	0	0	$(2x)^2 = 2x$
$x = 0.25$	0.25	0.5	$(2x)^2 < 2x$

it is not always true for values less than 0 and greater than $\frac{1}{2}$

17 (a) Write down the exact value of $\tan 60^\circ$.

(a) $\sqrt{3}$ [1]

(b) Find the exact area of this triangle.



$$\tan 60 = \frac{0}{4\sqrt{3}}$$

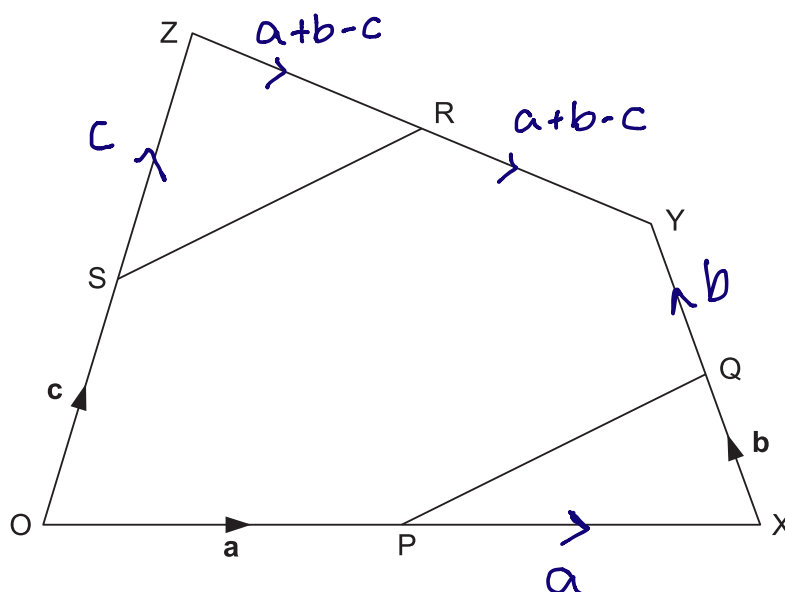
$$0 = \sqrt{3} \times 4\sqrt{3}$$

$$= 4 \times 3 = 12 \text{ cm}$$

$$\text{Area} = \frac{4\sqrt{3} \times 12}{2} = 24\sqrt{3}$$

(b) $24\sqrt{3}$ cm^2 [4]

18 P, Q, R and S are the midpoints of OX, XY, YZ and OZ respectively.



$$\vec{OP} = \mathbf{a}, \vec{XQ} = \mathbf{b} \text{ and } \vec{OS} = \mathbf{c}.$$

Show that PQ is parallel to SR.

[5]

$$\vec{ZY} = -2\mathbf{c} + 2\mathbf{a} + 2\mathbf{b}$$

$$\vec{ZR} = \mathbf{a} + \mathbf{b} - \mathbf{c}$$

$$\begin{aligned} \vec{SR} &= \mathbf{c} + \mathbf{a} + \mathbf{b} - \mathbf{c} \\ &= \mathbf{a} + \mathbf{b} \end{aligned}$$

$$\vec{PQ} = \mathbf{a} + \mathbf{b}$$

so $\vec{PQ} = \vec{SR}$ so they are parallel

19 The prices of two phones are in the ratio $x : y$.

When the prices are both increased by £20, the ratio becomes 5 : 2.

When the prices are both reduced by £5, the ratio becomes 5 : 1.

Express the ratio $x : y$ in its lowest terms.

$$\begin{array}{l} \text{original} \quad x : y \\ \uparrow \quad x + 20 : y + 20 = 5 : 2 \\ \downarrow \quad x - 5 : y - 5 = 5 : 1 \end{array}$$

$$\frac{x+20}{y+20} = \frac{5}{2} \quad 2(x+20) = 5(y+20)$$

$$2x + 40 = 5y + 100$$

$$2x - 5y = 60$$

$$\frac{x-5}{y-5} = \frac{5}{1} \quad x-5 = 5(y-5)$$

$$x-5 = 5y-25$$

$$\begin{array}{r} x - 5y = -20 \\ \hline x \quad x = 80 \end{array}$$

$$\begin{array}{l} 80 - 5y = 20 \\ 5y = 60 \\ y = 12 \end{array}$$

Ratio $x : y$

$$80 : 20$$

$$4 : 1$$

$$\dots 4 : 1 \dots [6]$$

- 20 (a) Find the interval for which $x^2 - 7x + 10 \leq 0$.

$$(x - 2)(x - 5) \leq 0$$

$$x \geq 2 \quad x \leq 5$$

(a) 2 $\leq x \leq$ 5 [3]

- (b) The point (-3, -4) is the turning point of the graph of $y = x^2 + ax + b$, where a and b are integers.

Find the values of a and b .

$$y = (x + 3)^2 - 4$$

$$= x^2 + 6x + 9 - 4$$

$$= x^2 + 6x + 5$$

(b) $a =$ 6 $b =$ 5 [3]

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