OCR
Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics
J560/06 Paper 6 (Higher Tier)
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

worked solutions

You may use:
- A scientific or graphical calculator
- Geometrical instruments
- Tracing paper

JustMaths

First name	
Last name	

Centre number						**Candidate number**				

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Turn over

$D = \dfrac{m}{V}$ $V = \dfrac{m}{D}$

Answer **all** the questions

	Copper	Zinc
D	$9 g/cm^3$	$7 g/cm^3$
Alloy	180g	105g
Volume	20	15

1   180 g of copper is mixed with 105 g of zinc to make an alloy.

The density of copper is 9 g/cm^3.
The density of zinc is 7 g/cm^3.

(a) Work out the volume of copper used in the alloy.

Volume $= \dfrac{180}{9}$

(a) ....**20**......... cm^3 **[2]**

(b) What is the density of the alloy?

Volume Zinc $: \dfrac{105}{7} = 15$

Total V $= 35\,cm^3$
Total m $= 180 + 105 = 285g$

$D = \dfrac{285}{35} = 8.142857$

(b) ....**8·14**......... g/cm^3 **[4]**

2 (a) (i) Solve.

$$5x + 1 > x + 13$$

$-x - x$

$4x + 1 > 13$
$-1 \quad -1$

$\frac{4x}{4} > \frac{12}{4}$

(a)(i) ....$x > 3$........ [3]

(ii) Write down the largest integer that satisfies $5x - 1 < 10$.

$5x < 9$

$x < \frac{9}{5}$

$x < 1.8$

(ii) ....2........ [1]

(b) Solve.

$$3x^2 = 75$$

$x^2 = 25$

(b) $x =$ ....± 5........ [2]

(c) Solve.

$$4x + 3y = 5 \quad -①$$
$$2x + 3y = 1 \quad -②$$

$①-②$ $2x = 4$

$x = 2$

Sub into ②

$2 \times 2 + 3y = 1$

$3y = 1 - 4$
$= -3$

$y = -1$

(c) $x =$ ....2........

$y =$ ....-1........

[3]

3 **(a)** This expression can be used to generate a sequence of numbers.

$$n^2 - n + 11$$

(i) Work out the first three terms of this sequence.

$1^2 - 1 + 11$ $2^2 \cdot 2 + 11$ $3^2 - 3 + 11$

$4 - 2 + 11$ $9 - 3 + 11$

(a)(i) ......**11**...... , ...**13**...... , ...**17**...... **[2]**

(ii) Show that this expression does not only generate prime numbers.

when n = 11

$11^2 - 11 + 11 = 121 - 11 + 11 = 121$ $11 \times 11 = 121$ $1 \times 121 = 121$

121 is not prime **[2]**

(b) Marta says

✓ 1, 4, 9, 16, 25

odd square numbers have exactly three factors.

Give one example where this is correct and another where this is not correct.
In each case, write down the number and its factors.

Correct .....**9   1×9, 3×3   3 factors 1,3,9**.....

Not correct .....**1   only has 1 factor**.....

[2]

(c) Here are some properties of a number.

- It is a common factor of 288 and 360.
- It is a common multiple of 4 and 6. 12, 24, 36, 48, 60, 72
- It is larger than 25.

Find the **two** possible numbers with these properties.

factors of 288

1, 288
2, 144
3, 96
4, (72)
6, 48
8, (36)
9, 32
12, (24) ← < 25
16, (18) ← < 25
18, 16 ← repeated

360

1, 360
2, 180
3, 120
4, 90
5, (72)
6, 60
9, 40
10, (36)
12, 30
15, (24) < 25
25 > (18), 20
20, 18

(c) ......36...... and ......72...... **[4]**

4 Here are the interest rates for two accounts.

Account A	**Account B**
Interest: 3% per year compound interest.	Interest: 4% for the first year, 3% for the second year and 2% for the third year.
No withdrawals until the end of three years.	Withdrawals allowed at any time.

Derrick has £10 000 he wants to invest.

(a) Calculate which account would give him most money if he invests his money for 3 years.
Give the difference in the interest to the nearest penny.

A
10000×1.03^3
10927.27

B
10000×1.04
$\times 1.03$
$\times 1.02$
$= 10926.24$

$10927.27 - 10926.24$

(a) Account ...**A**... by ...**103**... p **[5]**

(b) Explain why he might **not** want to use Account A.

he may want to withdraw money at any point and A says withdrawals arent allowed **[1]**

5 Lei is in a class of 28 students, 3 of whom are left-handed.
There are 1250 students in the school.

(a) Use this information to estimate how many students in the school are left-handed.

$$\frac{3}{30} = \frac{1}{10}$$

$$\frac{1}{10} \times 1250$$

(a) .....125..... **[3]**

(b) Is your solution to **(a)** likely to be an overestimate or an underestimate?
Explain your reasoning.

It is likely to be an underestimate as $\frac{3}{30}$ is smaller than $\frac{3}{28}$

.... **[1]**

(c) Vid is at a different school.
He is in a class of 26 students, 6 of whom are left-handed.

Vid says to Lei

In our two classes there are 54 students, 9 of whom are left-handed.
We can use this bigger sample to improve the estimate for your school.

What assumption has Vid made?
Explain whether you think that his argument is correct.

he has assumed the proportions of left handers at both schools are
both the same. No I don't think his argument is correct. The **[2]**
two samples are from different populations

6 John wants to investigate whether men in the UK are better at estimating a time interval of 10 seconds than women in the UK. He decides to sample the population by asking his work colleagues to take the test.

The diagrams below summarise John's results.
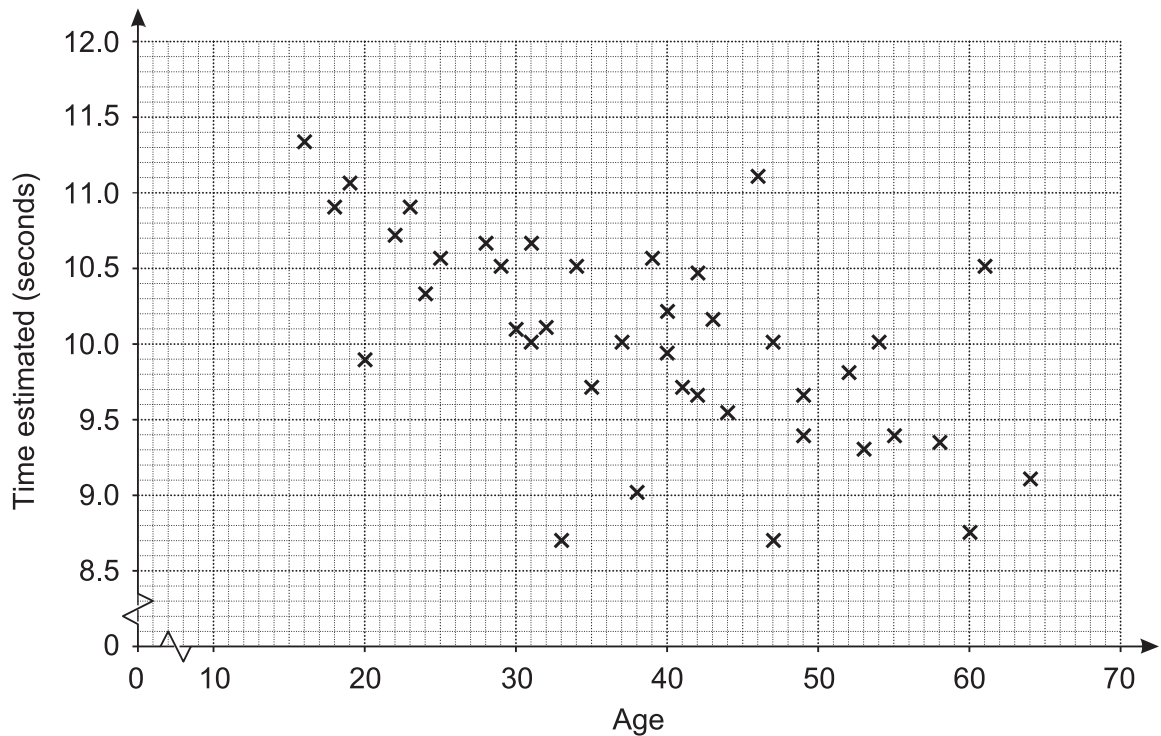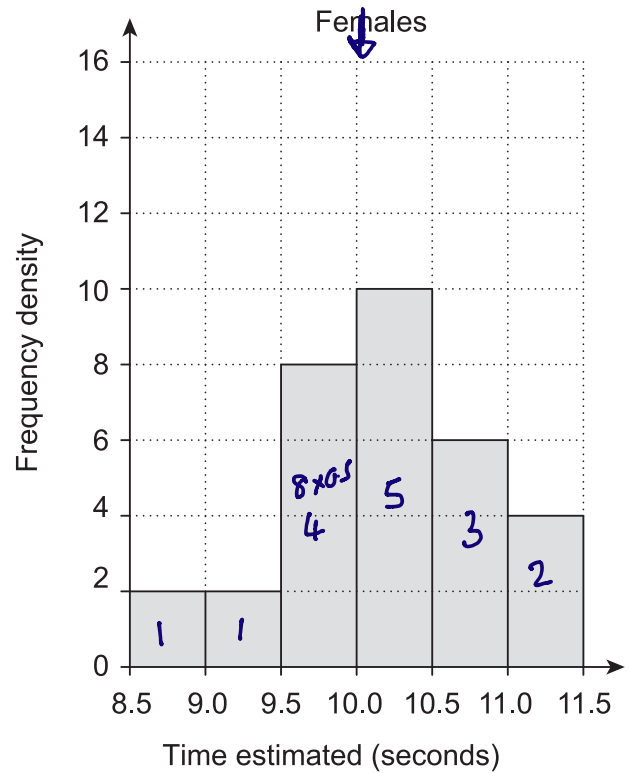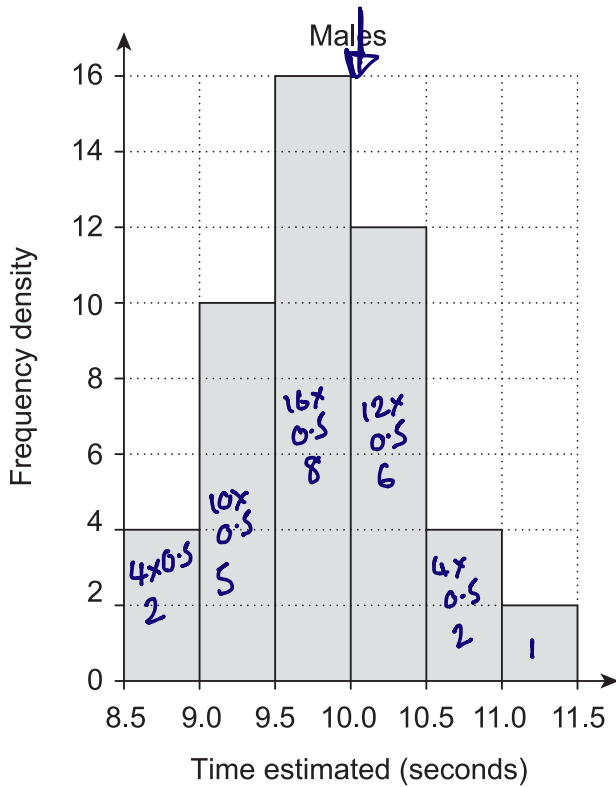
(a) What information from the diagrams can be used to support each of these statements?

    (i) The older John's colleagues are, the lower their estimate is.

       the scatter graph.

       It has a negative correlation **[1]**

    (ii) Males in the sample tend to underestimate the interval and females in the sample tend to overestimate the interval.

       Histograms: Men $\frac{15}{24}$ have times less than 10s $\frac{15}{24} = 62.5\%$

       Women $\frac{10}{16}$ have times more than 10s 62.5% **[2]**

(b) Comment on whether any conclusions can be drawn for the UK population from the results of this sample.

    The sample of 40 is too small to make inferences about the UK

    population so no **[2]**

7 Without using a calculator, show clearly that $64^{\frac{2}{3}}$ is equal to 16. **[2]**

    $64^{2/3}$ $\sqrt[3]{64} = 4$ $(4 \times 4 \times 4 = 64)$

                          $4^2 = 16$

8 **(a)** Prove that the sum of four consecutive whole numbers is always even. **[3]**

let n = first whole number

n+1
n+2
n+3

$4n+6 = 2(2n+3)$

$2n+3$ could be odd or even

$2(2n+3)$ ← always even

(b) Give an example to show that the sum of four consecutive integers is **not** always divisible by 4.

$1 + 2 + 3 + 4 = 10$

so $2(2n+3)$ isnt always divisible by 4

.... **[2]**

9 Alexander, Reiner and Wim each watch a different film.

- Alexander's film is thirty minutes longer than Wim's film.
- Reiner's film is twice as long as Wim's film.
- Altogether the films last 390 minutes.

How long is each of their films?

$$A = W + 30m$$
$$R = 2W$$
$$W = W$$

$$A + R + W = 390$$
$$W + 30 + 2W + W = 390$$
$$4W + 30 = 390$$
$$4W = 360$$
$$W = 90$$

Alexander's film ....120............ minutes

Reiner's film ....180............ minutes

Wim's film ....90............ minutes

[4]

10 The graph shows the distance travelled by an animal over 12 seconds.
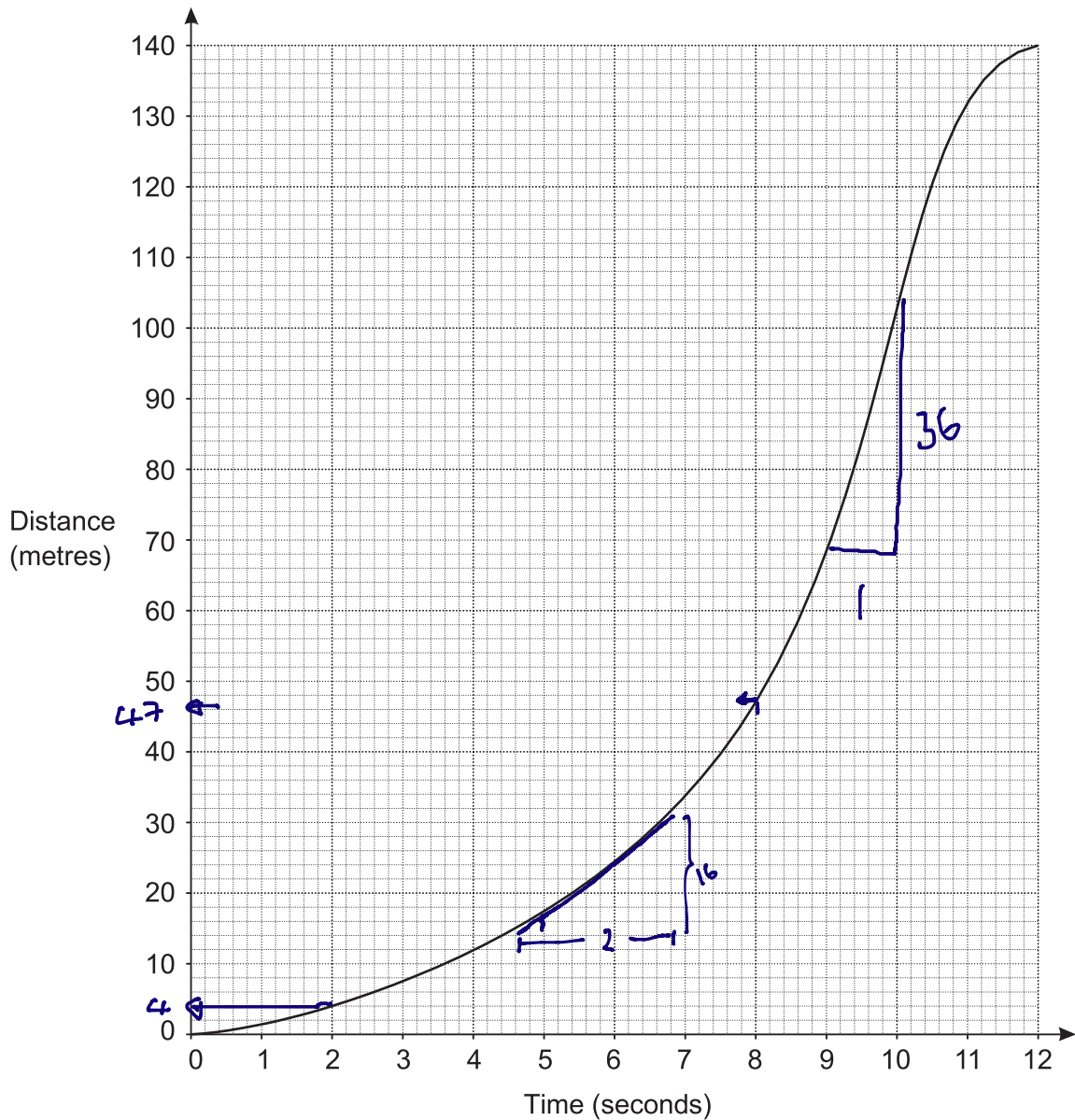
(a) Work out the average speed between 2 and 8 seconds.

$t = 2 \quad\quad 8 \quad\quad\quad \boxed{6}$

$D = 4 \quad\quad 47 \quad\rightarrow \boxed{43}$

$S = \dfrac{43}{6} = 7.16$

(a) ....7.2............ m/s **[2]**

(b) Estimate the speed of the animal at 6 seconds.

$\dfrac{16}{2}$

(b) ....8............ m/s **[4]**

(c) Nuri says

> I think this animal must be able to move at over 20 m/s!

Do you agree with Nuri?
Explain your decision.

Yes, between 9 and 10 seconds it moves 36m which means a speed of ≃ 36m/s > 20m/s **[2]**

11 A skills test has two sections, literacy (L) and numeracy (N).
One day everyone who took the skills test passed at least one section.
88% passed the literacy section and 76% passed the numeracy section.

12% numeracy   24 lit only

(a) Represent this information on a Venn diagram.
Show clearly the **percentage** in each section of the diagram.
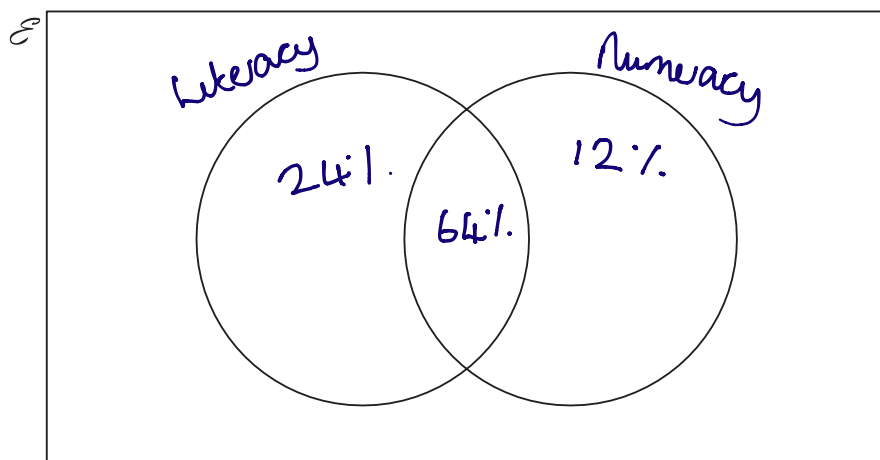
[3]

(b) One person is chosen at random from all the people who took the skills test that day.

What is the probability that this person

(i) passed the numeracy section, given that they passed the literacy section,

(b)(i) $\dfrac{64}{88}$ **[2]**

(ii) passed the literacy section, given that they passed only one section?

(ii) $\dfrac{24}{36}$ **[2]**

12 Two similar pyramids A and B have surface areas 180 cm^2 and 80 cm^2 respectively.

180 ⟶ 80

area SF

$= \dfrac{9}{4}$

÷
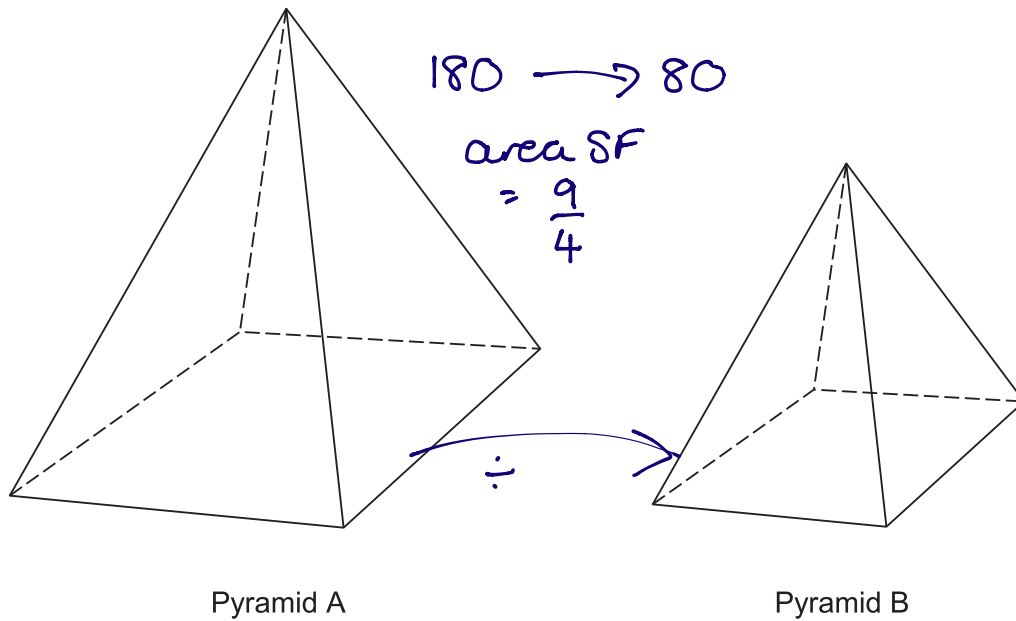
Pyramid A   Pyramid B

The volume of pyramid A is 810 cm^3.

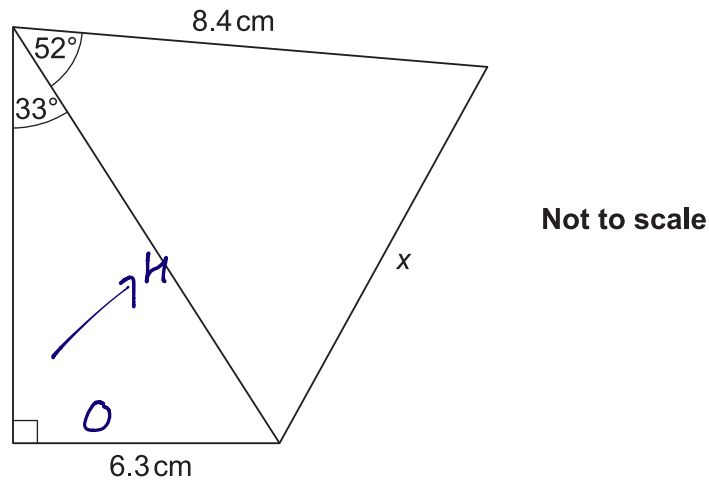Show that the volume of pyramid B is 240 cm^3.   **[5]**

Length SF $= \sqrt{\dfrac{9}{4}} = \dfrac{3}{2}$

Volume SF $= \dfrac{3^3}{2^3} = \dfrac{27}{8}$

$810 \div \dfrac{27}{8} = 240 \text{cm}^3$

as required

13 Calculate x.

8.4 cm
52°
33°
H
O
6.3 cm
x

Not to scale

$\sin 33 = \dfrac{6.3}{H}$ $H = \dfrac{6.3}{\sin 33} = 11.567...$

$a^2 = b^2 + c^2 - 2bc \cos A$

$= 8.4^2 + 11.567^2 - 2 \times 8.4 \times 11.567 \times \cos 52$

$= 84.7235$

$a = \sqrt{84.7235}$

$= 9.20453$

9.20 .... cm **[5]**

100000[]

14 A straight line goes through the points (p, q) and (r, s), where

- $p + 2 = r$ $p = r - 2$
- $q + 4 = s.$ $q = s - 4$
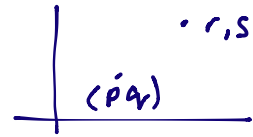
Find the gradient of the line.

$$\text{gradient} : \frac{s - q}{r - p} = \frac{q + 4 - q}{p + 2 - p} = \frac{4}{2}$$

2 .... **[3]**

15 A unit fraction has a numerator equal to 1, for example $\frac{1}{3}$, $\frac{1}{7}$ and $\frac{1}{25}$.

Unit fractions can be written as the sum of two different unit fractions, for example

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$$

Write each of the following unit fractions as the sum of two **different** unit fractions.

$$\frac{1}{4} = \frac{1}{\boxed{12}} + \frac{1}{\boxed{6}}$$ $\frac{2}{12} = \frac{3}{12}$

$$\frac{1}{5} = \frac{1}{\boxed{6}} + \frac{1}{\boxed{30}}$$

$$\frac{1}{6} = \frac{1}{\boxed{9}} + \frac{1}{\boxed{18}}$$

[3]

16 Simon cuts the corners off a square piece of card to leave the regular octagon shown below.
O is the centre of the octagon.
A and B are vertices of the octagon.
OA = OB = 5 cm.
Angle AOB = 45°.
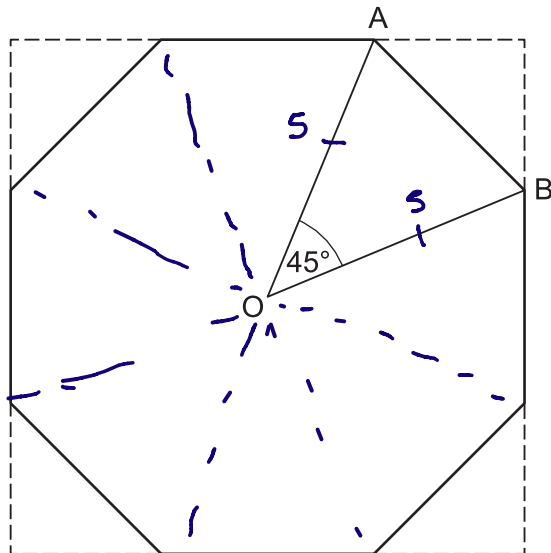
Not to scale

(a) (i) Work out the area of the octagon.

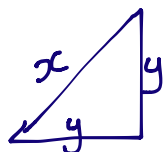$$\left(\frac{1}{2} \times 5 \times 5 \times \sin 45\right) \times 8$$

$$= 70.710678$$

(a)(i) ....70.71............ cm² **[3]**

(ii) Work out the area of the original square piece of card.

S
x $135 \div 2$
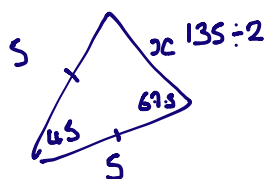
$\dfrac{5}{\sin 67.5} = \dfrac{x}{\sin 45}$

$x = \dfrac{5}{\sin 67.5} \times \sin 45 = 3.826834324$

$3.82^2 = y^2 + y^2$

$2y^2 = 14.644 \qquad y = 2.705980501$

length of side $= 2.705980501 \times 2 + 3.82$

Area $= 9.23879 \times 9.23879$

$= 85.35533907$

(ii) .... 85.4 .... cm^2 **[5]**

(b) Simon now makes a table top using the card as a model.
The sides of the table top are 8 times as long as the sides of the card model.

Find the ratio of the **area** of Simon's table top to the **area** of the card model.

Length SF $= 8$

area SF $= 8^2$

(b) .... 1 .... : .... 64 .... **[2]**

17 $y = 6x^4 + 7x^2$ and $x = \sqrt{w+1}$. $x^2 = w + 1$

$x^4 = (x^2)^2 = (w+1)^2$
$= w^2 + 2w + 1$

Find the value of w when $y = 10$.
Show your working.

$10 = 6x^4 + 7x^2$

$0 = 6(w^2 + 2w + 1) + 7(w+1) - 10$

$6w^2 + 12w + 6 + 7w + 7 - 10 = 0$

$6w^2 + 19w + 3 = 0$

⑱

$1,18$

$6w^2 + 18w + 1w + 3 = 0$

$6w(w+3) + 1(1w+3) = 0$

$(6w+1)(w+3) = 0$

$w = -\frac{1}{6} \qquad w = -3$

$w = -3 \qquad x = \sqrt{-3+1} \qquad$ NOT A SOLUTION FOR x SO

NOT A VALID SOLUTION FOR w

$w = \dfrac{-1/6}{} $ .... **[6]**