OCR
Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Practice Paper

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

You may use:
- A scientific or graphical calculator
- Geometrical instruments
- Tracing paper

First name	JustMaths
Last name	Worked Solutions
Centre number	
Candidate number	

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions

1 **(a)** The attendance at a football match was 67 500, correct to the nearest hundred.

(i) What was the **highest** possible attendance?

67,549

67 500

67,450

(a)(i) 67,549 **[1]**

(ii) What was the **lowest** possible attendance?

this is key.

(ii) 67,450 **[1]**

(b) A distance, d, was given as 6.73 m, **truncated** to 2 decimal places.

Complete the error interval for the distance, d.

6·73 $\leqslant d <$ 6·74 **[2]**

2 The population, P, of an island t years after January 1st 2016 is given by this formula.

$$P = 4200 \times 1.04^t$$

(a) What was the population of the island on January 1st 2016?

$t:0$

$P = 4200 \times 1.04^0$

(a) 4200 **[1]**

(b) Explain how you know that the population is increasing.

The percentage multiplier is > 1 as 1.04 is adding 4%.

[1]

(c) What is the annual percentage increase in the population?

(c) 4 % **[1]**

(d) Work out the population of the island on January 1st 2021.

2016
17 1
18 2
19 3
20 4
21 5

$P = 4200 \times 1.04^5$
$= 5109.94219$

(d) 5110 **[2]**

3 A shop has a sale that offers 20% off all prices.
On the final day they reduce all sale prices by 25%.
Alex buys a hairdryer on the final day.

Work out the **overall** percentage reduction on the price of the hairdryer.

let original = £100

sale price = 100×0.8 = £80

final day = 80×0.75 = £60

100 → 60 = £40 off

which is $\frac{40}{100}$ = 40% reduction

40 ..... % **[6]**

4 An interior angle of a regular polygon is eleven times its exterior angle.

Work out the number of sides of the polygon.

11x / x

$12x = 180$

$x = \frac{180}{12}$

$= 15^{\circ}$ = 1 exterior angle

No. of sides = $360 \div 15$

= 24 sides

24 **[4]**

5 **(a)** Find the *n*th term of this linear sequence.

8 , 11 , 14 , 17

(+3) (+3) (+3)

(a) 3n+5 **[2]**

(b) Here is a quadratic sequence.

2 14 36 68

The expression for the *n*th term of this sequence is $pn^2 + qn$.

Find the value of *p* and the value of *q*.

n =	1	2	3	4
1st diff	2	14	36	68
2nd diff		12	22	32
			10	10

10 ÷ 2 = 5

	1	2	3	4
$5n^2$	5	20	45	80
$5n^2 - n$	3			
$3n$	3	6	9	12
$5n^2 - 3n$	2	14	36	68

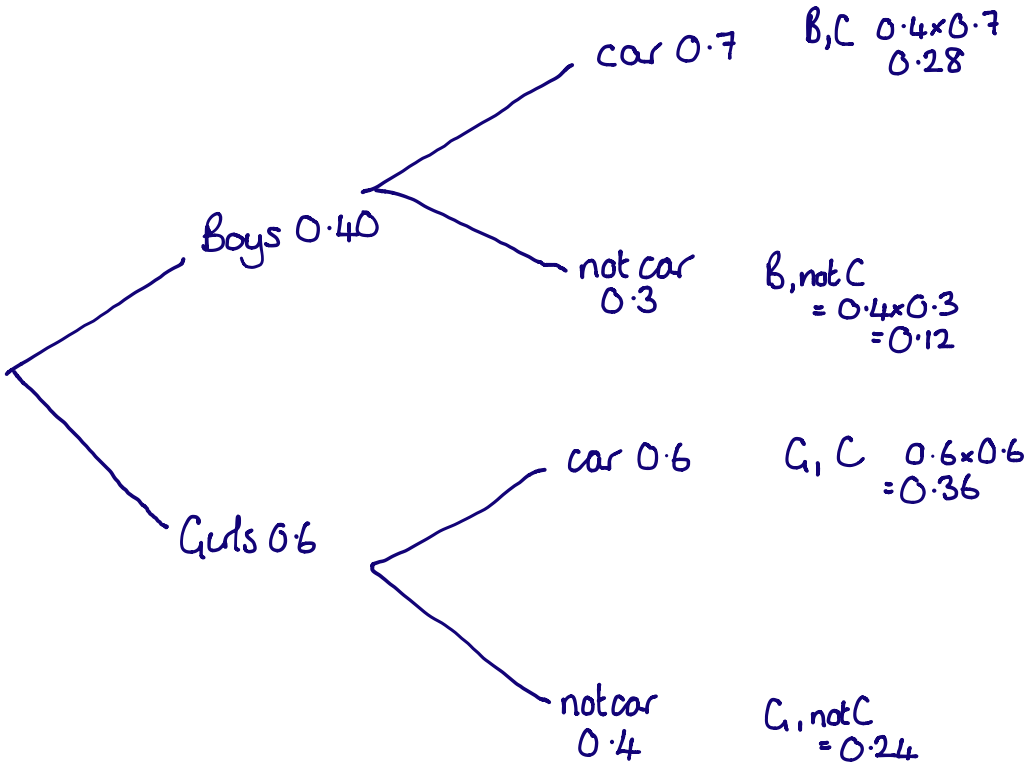
(b) *p* = 5

q = −3 **[4]**

6 Some of the children at a nursery arrive by car.

- 40% of the children at the nursery are boys.
- 70% of the boys at the nursery arrive by car.
- 60% of the girls at the nursery arrive by car.

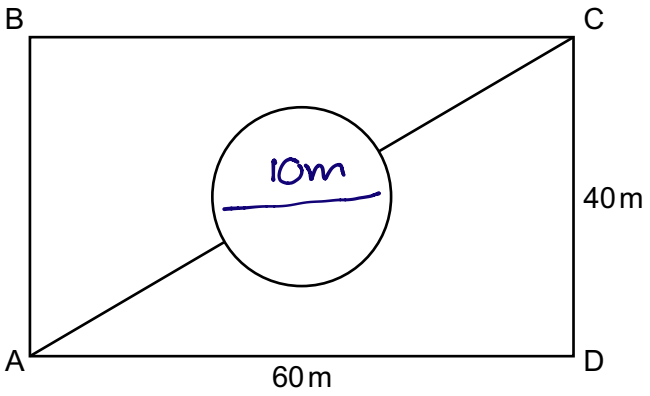
What is the probability that a child chosen at random from the nursery arrives by car?



0·28 + 0·36

0·64 ..... **[5]**

7 The rectangle ABCD represents a park.



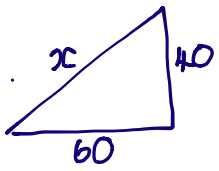
Not to scale

The lines show all the paths in the park.
The circular path is in the centre of the rectangle and has a diameter of 10 m.

Calculate the shortest distance from A to C across the park, using only the paths shown.

Circumference of circle = $\pi \times 10 = 31.41592$ m

half circle = $5\pi = 15.70796$ m



$x^2 = 40^2 + 60^2$

$x = \sqrt{5200} = 20\sqrt{13}$

Shortest distance $= 20\sqrt{13} - 10 + 5\pi$

$= 77.81898878$

77.82 (2dp) ..... m **[6]**

8 Eddie and Caroline are going to the school play.

Eddie buys 6 adult tickets and 2 child tickets. He pays £39.
Caroline buys 5 adult tickets and 3 child tickets. She pays £36.50.

Work out the cost of an adult ticket and the cost of a child ticket.

$6a + 2c = 39$ -①
$5a + 3c = 36.50$ -②

① × 3
② × 2

$18a + 6c = 117$ -③
$10a + 6c = 73$ -④

③ - ④

$8a = 44$
$a = £5.50$

sub into ② $5 \times (5.50) + 3c = 36.50$
$27.50 + 3c = 36.50$
$3c = 36.50 - 27.50$
$3c = 9$
$c = £3$

Adult ticket £ 5.50

Child ticket £ 3.00 **[5]**

9 Gavin measures the heights of 80 plants he has grown.
This table summarises his results.

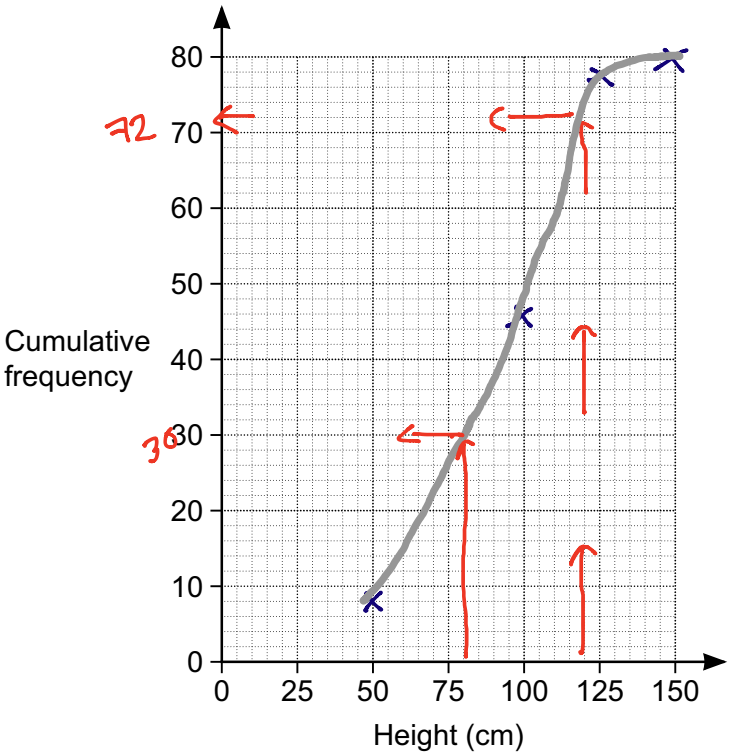
Height, h cm	$0 < h \leqslant 50$	$50 < h \leqslant 100$	$100 < h \leqslant 125$	$125 < h \leqslant 150$
Number of plants	8	38	31	3

(a) **(i)** Complete the cumulative frequency table below.

Height, h cm	$h \leqslant 50$	$h \leqslant 100$	$h \leqslant 125$	$h \leqslant 150$
Cumulative frequency	8	46	77	80

[2]

(ii) Draw the cumulative frequency graph.



[2]

(b) Ted asks if Gavin has 10 plants over 120 cm in height.

Explain why Gavin cannot be certain that he has 10 plants over this height.

The exact measurements aren't noted

..... **[1]**

(c) Gavin sells these 80 plants using the price list below.

Height, h cm	$h \leqslant 80$	$80 < h \leqslant 120$	$h > 120$
Price (£)	2.00	3.50	5.00

30 72 - 30 = 42 80 - 72 = 8

Each plant costs him 60p to grow.

Estimate the total profit Gavin will receive when he sells all these plants.

30 x £2.00 = £60
42 x 3.50 = £147
8 x 5.00 = £40

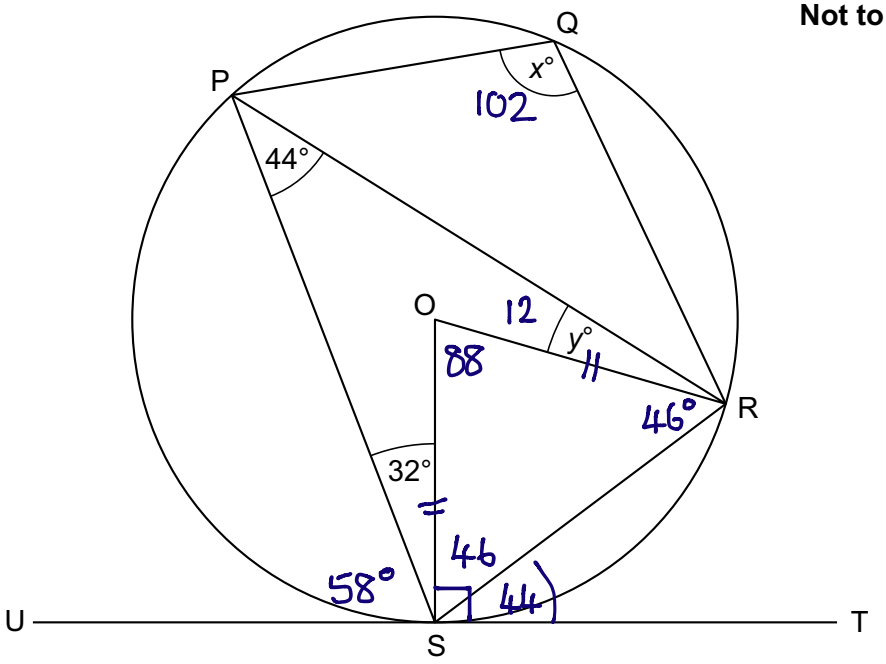
Total £247

Costs = 80 x 0.60 = £48

£247 - £48
= £199

(c) £ 173 **[6]**

10 The diagram shows a circle, centre O.
Points P, Q, R and S lie on the circumference of the circle.
UST is a tangent to the circle.
Angle RPS = 44° and angle PSO = 32°.



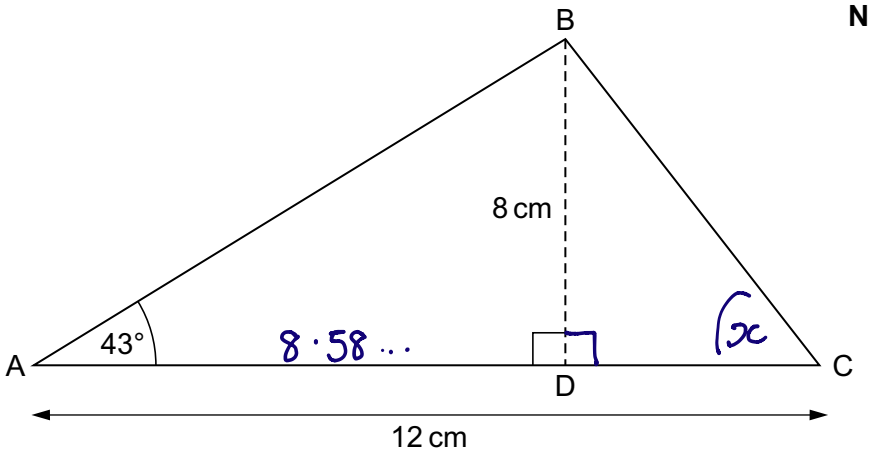
(a) Work out the value of x.

(a) x = 102 **[4]**

(b) Work out the value of y.

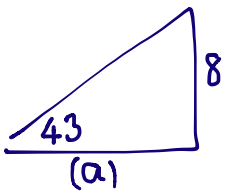
(b) y = 12 **[3]**

11 In the diagram, ABC is a triangle and line BD is perpendicular to AC.
Angle BAC = 43°, BD = 8 cm and AC = 12 cm.



Not to scale

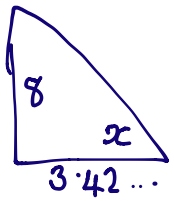
Calculate angle BCA.



$\tan 43 = \frac{8}{a}$

$a = \frac{8}{\tan 43} = 8.57894968$

$\therefore DC = 12 - 8.58... = 3.42105032$



$\tan x = \frac{8}{3.42}$

$= 2.338...$

$x = \tan^{-1}(2.338...)$

$= 66.84693243$

66.85 ..... ° **[6]**

12 Show that $k = \frac{4+3j}{5-j}$ can be rearranged to $j = \frac{5k-4}{3+k}$. **[4]**

$k(5-j) = 4+3j$

$5k - kj = 4+3j$

$5k - 4 = 3j + kj$

$j(3+k) = 5k - 4$

$j = \frac{5k-4}{3+k}$ QED

13 **(a)** y is directly proportional to $\sqrt{x}$.
y is 75 when $x = 100$.

Find a formula linking x and y.

$y \propto \sqrt{x}$

$y = kx$

$x = 100$
$y = 75$

$75 = k \times \sqrt{100}$

$k = \frac{75}{10} = 7.5$

$y = 7.5\sqrt{x}$

(a) ..... **[3]**

(b) y is inversely proportional to x^2 and $y = 3$ when $x = 12$.

Show that $y = 27$ when $x = 4$. **[3]**

$y \propto \frac{1}{x^2}$ $\quad y = \frac{k}{x^2}$

$y = 3 \quad x = 12 \quad 3 = \frac{k}{12^2} \quad k = 3 \times 12^2 = 432$

$\therefore y = \frac{432}{x^2}$

when $x = 4$ $\quad y = \frac{432}{16} = 27$ QED

14 **(a)** Write $x^2 + 10x + 29$ in the form $(x + a)^2 + b$.

$(x+5)^2 - 25 + 29$

$(x+5)^2 + 4$

(a) $(x+5)^2 + 4$ **[3]**

(b) Write down the coordinates of the turning point of the graph of $y = x^2 + 10x + 29$.

(b) (-5 , 4) **[1]**

15 **(a)** Complete the table for $y = x^3 - 6x - 5$.

x	0	1	2	3	4
y	-5	-10	-9	4	35

[2]

(b) **(i)** Between which two **consecutive integers** is there a solution to the equation $x^3 - 6x - 5 = 0$?
Give a reason for your answer.

A solution lies between $x =$ 2 and $x =$ 3

because when $x = 2$ y is < 0 and when $x = 3$ $y > 0$

[2]

(ii) Choose a value of x between the two values you gave in part **(b)(i)**.
Calculate the corresponding value of y.

$x =$
2.5

$y = 2.5^3 - 6 \times 2.5 - 5$
$= -4.375$

(b)(ii) $x =$ 2.5

$y =$ -4.375 **[2]**

(iii) State a smaller interval in which the solution lies.

(iii) $2.5 < x < 3$ **[1]**

16 Solve these simultaneous equations algebraically.

$y = x - 3$ - ①

$y = 2x^2 + 8x - 7$ - ②

① = ②

$x - 3 = 2x^2 + 8x - 7$

$0 = 2x^2 + 7x - 4$ $\quad 2 \times 4 = 8$

$2x^2 + 8x - 1x - 4$

$2x(x + 4) - 1(x + 4)$

$(2x - 1)(x + 4) = 0$

$2x - 1 = 0$ or $x = -4$

$2x = 1$

$x = \frac{1}{2}$

$y = \frac{1}{2} - 3 = -2\frac{1}{2}$ $\quad y = -7$

$x =$ 0.5 , $y =$ -2.5

$x =$ -4 , $y =$ -7 **[6]**

17 **(a)** Show that $\sqrt{396}$ can be written as $6\sqrt{11}$. **[2]**

$\sqrt{396}$

$\sqrt{3} \times \sqrt{132}$

$\sqrt{3} \times \sqrt{3} \times \sqrt{44}$

$\sqrt{3}\sqrt{3} \times \sqrt{4}\sqrt{11}$

$\sqrt{3}\sqrt{3} \times 2\sqrt{11}$

$3 \times 2\sqrt{11} = 6\sqrt{11}$

(b) **Without** using a calculator, show that $\dfrac{4+2\sqrt{2}}{2-\sqrt{2}}$ can be simplified to $6+4\sqrt{2}$. **[6]**

$\dfrac{(4+2\sqrt{2}) \times (2+\sqrt{2})}{(2-\sqrt{2}) \times (2+\sqrt{2})} = \dfrac{8+4\sqrt{2}+4\sqrt{2}+2\times 2}{4+2\sqrt{2}-2\sqrt{2}-2}$

$= \dfrac{12+8\sqrt{2}}{2}$

$= \dfrac{12}{2} + \dfrac{8\sqrt{2}}{2}$

$= 6+4\sqrt{2}$ QED