Candidate Name	Centre Number	Candidate Number
JustMaths		0

GCSE MATHEMATICS

COMPONENT 1

Non-Calculator Mathematics

Higher Tier

SPECIMEN PAPER

2 hour 15 minutes

wjec cbac

eduqas

Worked Solutions

ADDITIONAL MATERIALS
The use of a calculator is not permitted in this examination.
A ruler, protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES
Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided in this booklet.

Take π as 3·14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the need for good English and orderly, clear presentation in your answers.

No certificate will be awarded to a candidate detected in any unfair practice during the examination.

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1.	3	
2.	4	
3.	4	
4.	6	
5.	3	
6.	4	
7.	4	
8.	4	
9.	3	
10.	5	
11.	4	
12.	5	
13.	7	
14.	6	
15.	5	
16.	10	
17.	4	
18.	4	
19.	2	
20.	5	
21.	7	
22.	6	
23.	7	
24.	8	
TOTAL	120	

Formula list

Area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of a cone } = \frac{1}{3}\pi r^2 h$$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

1. Given that $a = 10$, $b = 3$ and $c = -5$, find the value of each of the following expressions.

    (a) b^2 [1]

    $$3^2 = 9$$

    (b) $\dfrac{ab}{c}$ [1]

    $$\frac{10 \times 3}{-5} = \frac{30}{-5} = -6$$

    (c) $\dfrac{2bc}{a}$ [1]

    $$\frac{2 \times 3 \times -5}{10} = \frac{-30}{10} = -3$$

2. (a) Write $6 \cdot 8 \times 10^4$ in decimal notation. [1]

    68000

    (b) Write in standard form the value of $0 \cdot 0000853$. [1]

    $$8 \cdot 53 \times 10^{-5}$$

    (c) Find in standard form the value of $(3 \times 10^2) \times (5 \times 10^6)$. [2]

    $$3 \times 5 \times 10^2 \times 10^6$$
    $$15 \times 10^8$$
    $$1 \cdot 5 \times 10^9$$

3. Cherry Blossom paint is made by mixing red and white paint in a certain ratio.
4 litres of **red** paint is used to make 9 litres of Cherry Blossom paint.
The diagram below shows the relationship between the amount of red paint and the
amount of white paint needed to make Cherry Blossom paint.

Amount of red paint (litres)

2·5

2·0

1·5

1

0·5

0

Amount of white
paint (litres)

0   1   2   3
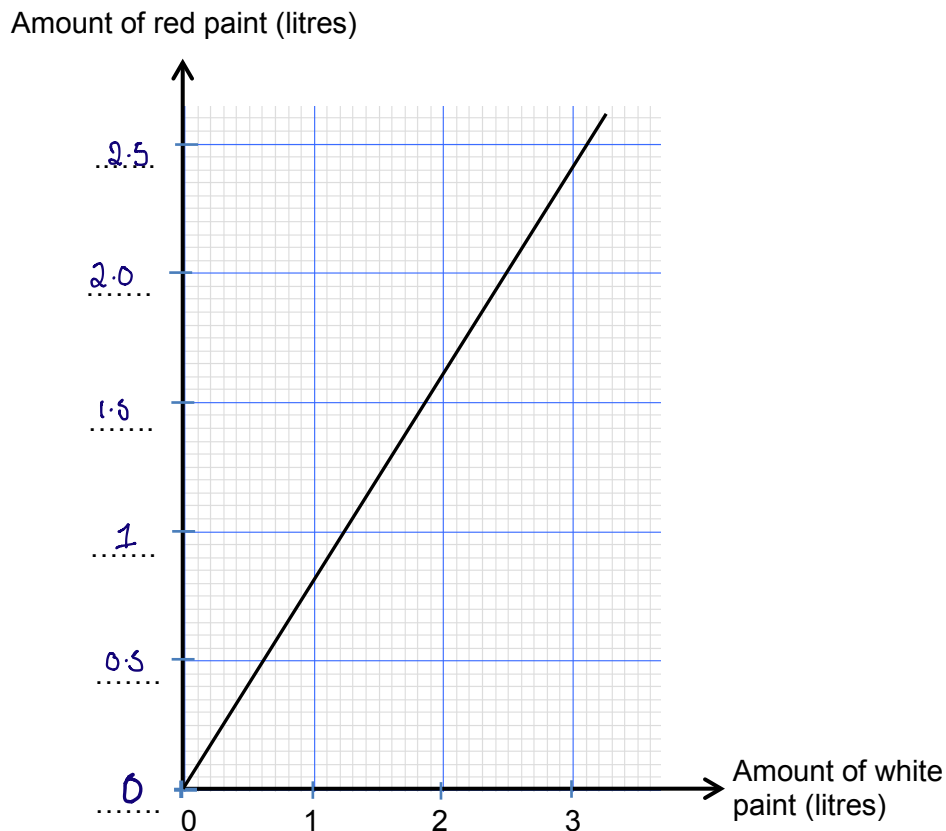
Write down the correct scale on the 'Amount of red paint (litres)' axis.
You must put a value on each of the dotted lines on the axis.
You must show all your working to support your answer.   [4]
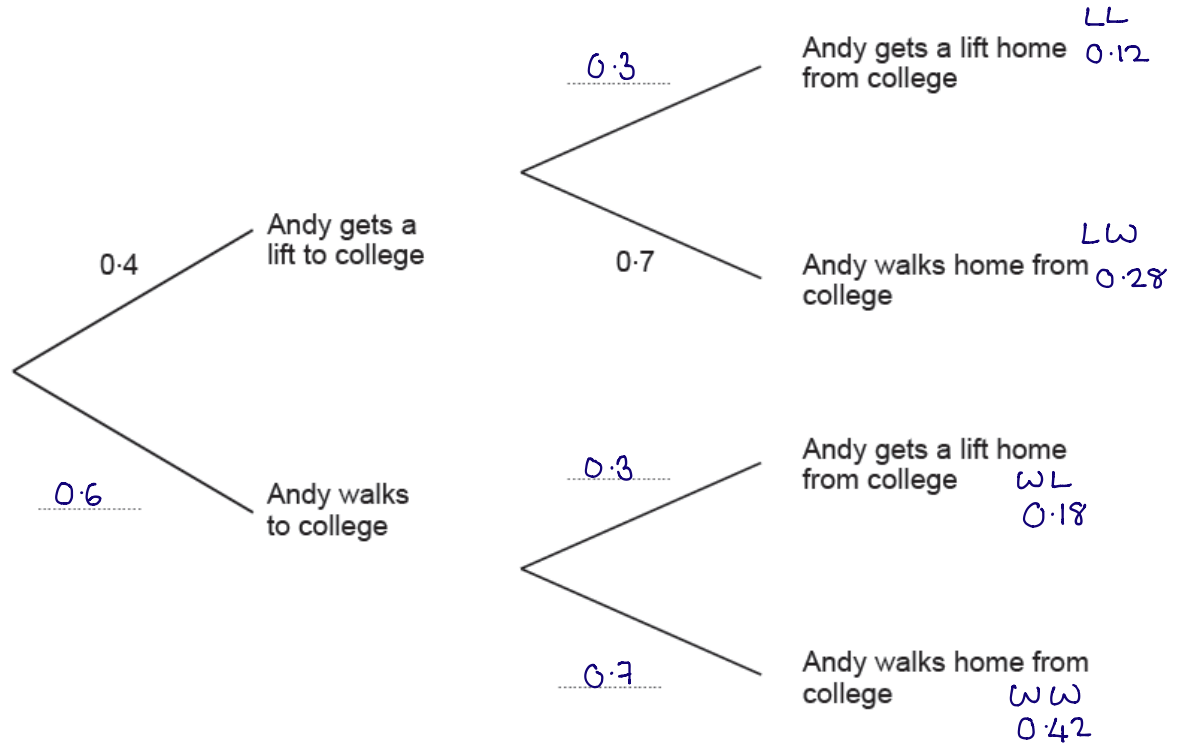
R : W

4 : 5

÷5 (4/5   1) ÷5

OR 0·8

4.   Andy sometimes gets a lift to and from college.
When he does not get a lift he walks.
The probability that he gets a lift to college is 0·4.
The probability that he walks home from college is 0·7.
Getting to college and getting home from college are independent events.

(a) Complete the following tree diagram.   [2]

LL
0·3   Andy gets a lift home   0·12
from college

0·4   Andy gets a
lift to college   0·7   Andy walks home from   LW
college   0·28

0·6   Andy walks
to college   0·3   Andy gets a lift home
from college   WL
0·18

0·7   Andy walks home from
college   WW
0·42

(b) Calculate the probability that Andy gets a lift to college and walks home from
college   [2]

0·4 × 0·7 = 0·28

(c) Calculate the probability that Andy **does not** get a lift to or from college.
[2]

0·6 × 0·7 = 0·42

5.   Write 3600 as a product of prime factors using index notation.   [3]

3600
360   10
36   10   ②   ⑤
6   6   ②   ⑤
② ③ ② ③
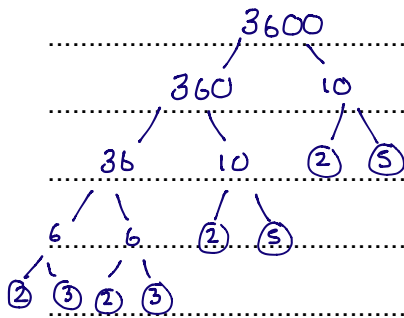
$$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$$
$$= 2^4 \times 3^2 \times 5^2$$

6. Alex bought 3 tins of paint and 4 brushes at a total cost of £23.
Brian bought 2 tins of paint and 3 brushes at a total cost of £16.

Using an algebraic method, calculate the price of a single tin of paint and the price of one brush. [4]

$$3t + 4b = 23 \quad -① \quad × ①$$
$$2t + 3b = 16 \quad -② \quad × ②$$

$$6t + 8b = 46 \quad -③$$
$$6t + 9b = 48 \quad -④$$

④ - ③ $\qquad b = 2$

sub into ②
$$2t + 3×2 = 16$$
$$2t + 6 = 16$$
$$2t = 10 \qquad t = 5$$

tin of paint = £5
1 brush = £2

The price of a single tin of paint = …… £5 ……………………

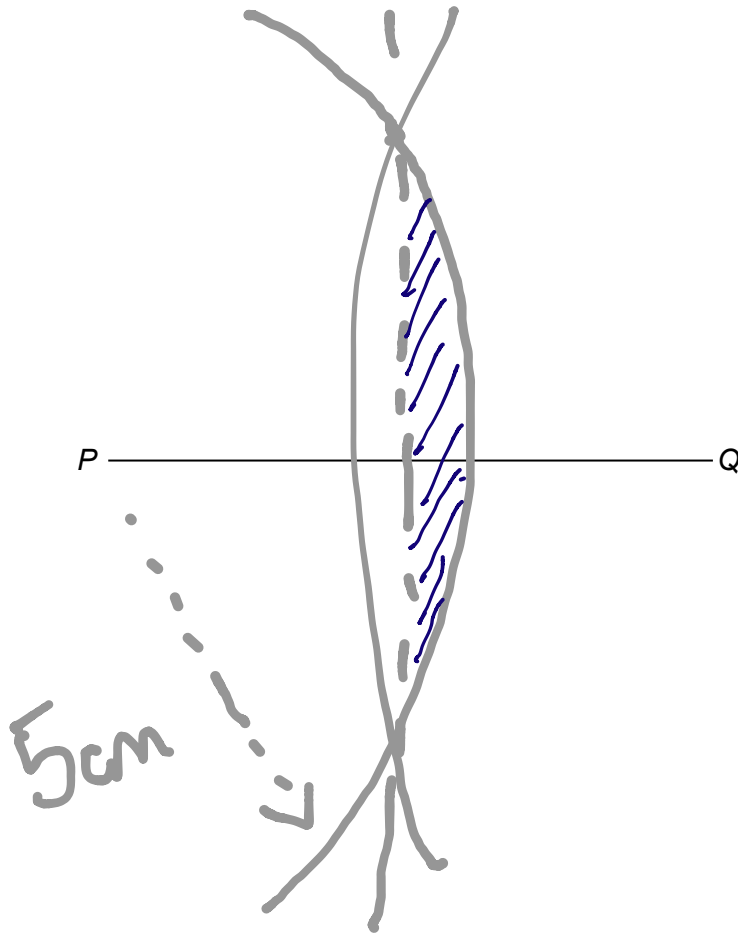The price of one brush = …… £2 ……………………..

7. Using ruler and compass construction, shade the region that satisfies both of the following conditions.

        (i)   The points are less than 5 cm from *P*

        (ii)   The points are nearer to *Q* than to *P*   [4]

NOT DRAWN TO SCALE!!

① an arc with a radius of 5cm

② a perpendicular bisector of PQ

P ———————————— *Q*

5cm

8. Two brothers, Richard and Andrew, share a sum of money in the ratio 2 : 7.
Andrew gets £30 more than Richard.
Calculate how much the brothers share. [4]

R : A

2 : 7

£30 more   £30 ÷ 5 = £6

2 × 6 : 7 × 6

12 : 42

Total = £54

9.   Factorise the following expressions.

(a) $6x^2 + 8x$ [2]

$$2x(3x+4)$$

(b) $x^2 - 100$ [1]

$$(x+10)(x-10)$$

10. Peter decides to cover the floor of a room with a striped carpet.
A shop sells this striped carpet from a roll that is 3 m wide at a price of £25 per metre length.
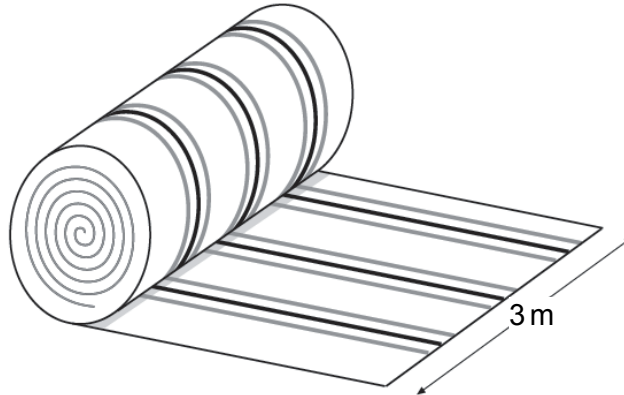
3 m

Diagram not drawn to scale

His floor is rectangular in shape with length 13 m and width 8 m.

8 m

= 40m
8
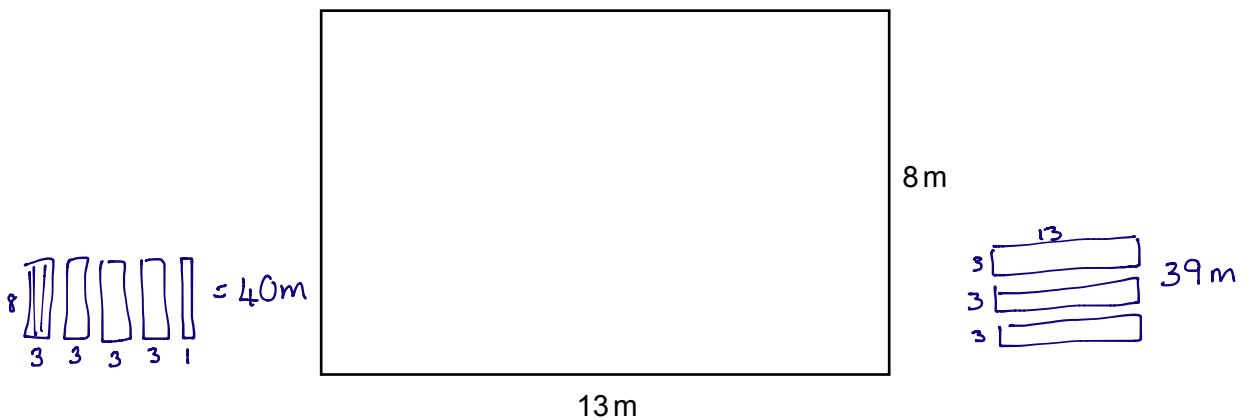3 3 3 3 1

13
3
3
3
39 m

13 m

Diagram not drawn to scale

The carpet is laid to ensure that the stripes on the carpet are parallel to two of the sides of the room and lie in one direction only.

Find the cost of the cheapest way of covering the floor, and state by how much it is cheaper.
Show all your working.   [5]

If the stripes are parallel to the longest side = 39m × £25   39 × 10 = 390
                                                                                                         5 = 195
                                                       = £975   × 10   390
                                                                                                            975

If stripes parallel to shortest side = 40m   40 × 25
                                                        = £1000

It is cheaper by £25 to lay the carpet parallel to the longest side

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11.   Vectors **DF** and **DE** are shown in the diagram below.

$$F$$

$$2\mathbf{a} + 5\mathbf{b}$$

$$D$$

$$3\mathbf{a} - 2\mathbf{b}$$

$$E$$
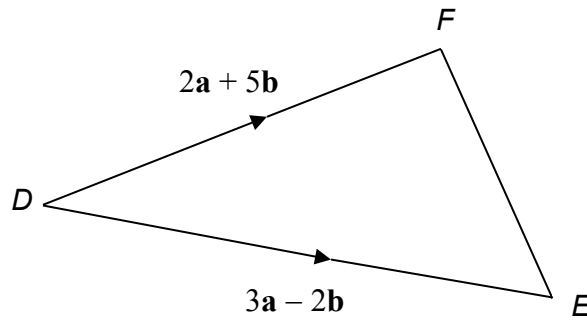
Line *PQ* is 3 times the length of line *EF*.
PQ is in the opposite direction to **EF**. $\boxed{FE}$

Find **PQ** in the form $m\mathbf{a} + n\mathbf{b}$.   [4]

$$\overrightarrow{FE} = -2a - 5b + 3a - 2b$$

$$= a - 7b$$

$$PQ = 3(a - 7b)$$

$$= 3a - 21b$$

12. A building company used 24 workers to prepare a building site.
The site measured 30 acres and the work was completed in 10 days.

(a) The company is asked to prepare another site measuring 45 acres.
This work has to be completed in 15 days.
Calculate the least number of workers the company should employ
for this work. [3]

24 workers => 30 acres => 10 days
30 ÷ 10 = 3 acres a day   24/3 = 8 workers/acre/day
45 acres => 15 days
45 ÷ 15 = 3 acres per day
so still 8 workers per acre.
so 24 workers are required

(b) State one assumption you have made in your answer to part *(a)*.
How would your answer to part *(a)* change if you did not make this
assumption? [2]

The work rate remains the same. If it is different it may take
more or less time

13. The line L_1 is shown in the diagram below.
The line L_2, which is not shown, is perpendicular to L_1.
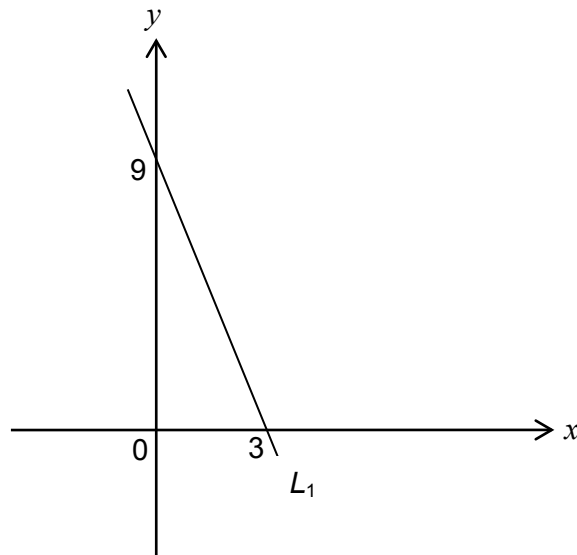
Diagram not drawn to scale

(a) *(i)* Find the gradient of L_1. [2]

$$\text{change in } y = \frac{-9}{3} = -3$$
$$\text{change in } x$$

(ii) Write down the gradient of L_2. [1]

$$\text{gradient } L_2 = \frac{1}{3}$$

(b) The two straight lines L_1 and L_2 intersect at the point (1, 6).
Find the equation of L_2 and write it in the form $ax + by + c = 0$. [4]

$$y = \frac{1}{3}x + c \qquad \text{at } (1,6) \qquad 6 = \frac{1}{3} \times 1 + c$$
$$c = 5\frac{2}{3}$$
$$y = \frac{1}{3}x + 5\frac{2}{3} \qquad y = \frac{x}{3} + \frac{17}{3}$$

$$3y = x + 17$$
$$3y - x - 17 = 0$$
$$x - 3y + 17 = 0$$

14. On a journey from Dover to Sheffield, Liam drove at an average speed of 40 mph for the first three hours of his journey.

The remaining 120 miles of his journey were completed at an average speed of 30 mph.

Liam told his friend that he had completed the whole journey at an average speed of 35 mph.

Check to see if Liam is correct. [6]

3 hours ← → 4 hours 35 × 7 - 35⁄7

40mph 30mph 245⁄7

120 miles ← 120 miles → 0.34.28

  7 √240.⁵0⁶0

$S = \dfrac{D}{T} = \dfrac{240}{7}$

= 34.3 mph. so Liam is not correct

15. Triangle *ABC* is isosceles with *AB = AC*.

The line *BP* bisects $A\hat{B}C$.

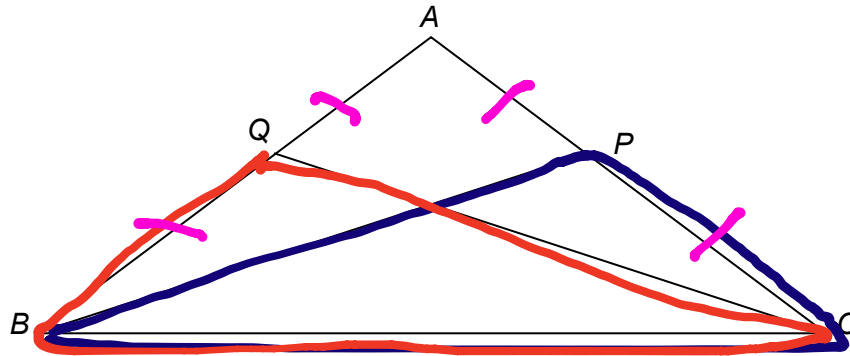The line *CQ* bisects $A\hat{C}B$.

Diagram not drawn to scale

Prove that triangle *BCP* and triangle *CBQ* are congruent.
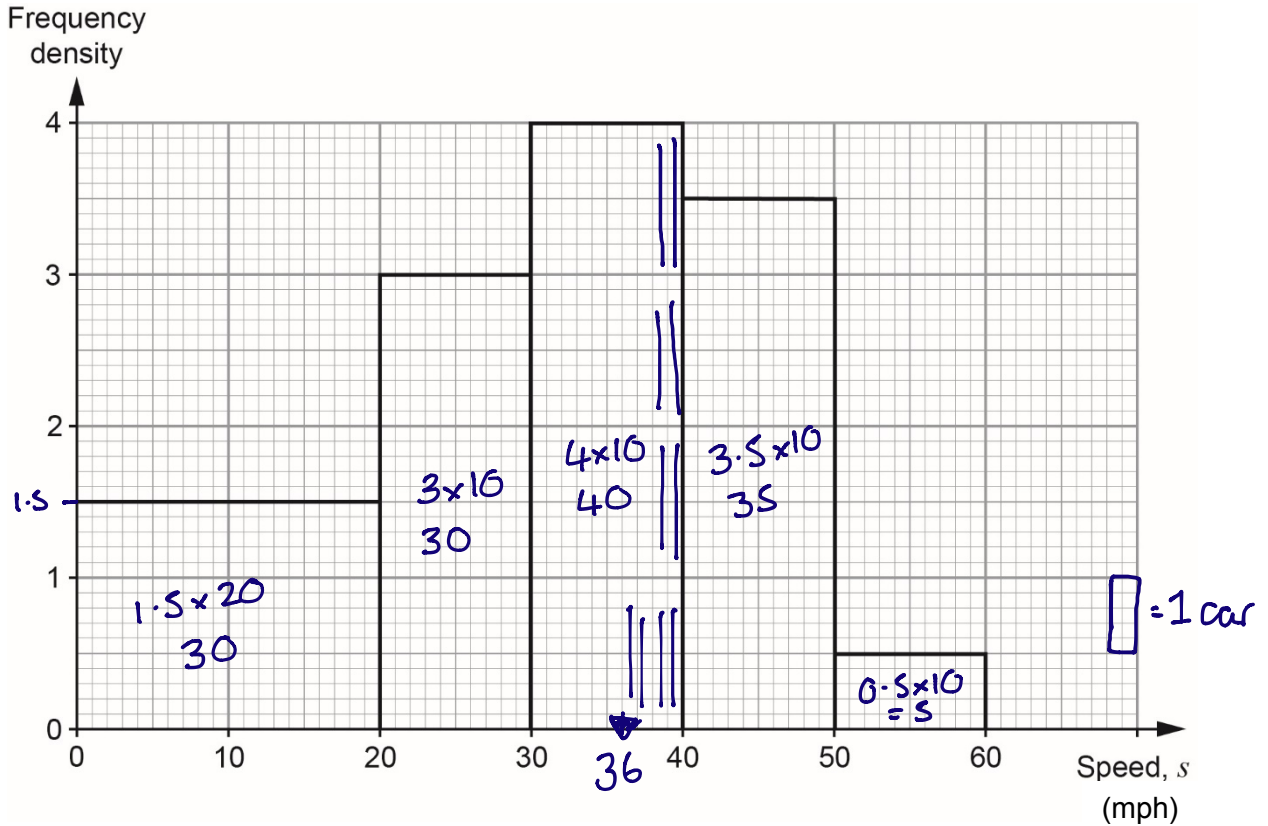You must give reasons to support your statements. [5]

$A\hat{B}C = A\hat{C}B$ 2 angles in an isosceles triangle are equal

BC = shared side

$P\hat{B}C = Q\hat{C}B$ angles were bisected

so triangles are congruent (ASA)

16. A survey was carried out to record the speeds of cars entering a village.
The histogram illustrates the results of the survey.

(a) Use the histogram to complete the grouped frequency table below. [2]

Speed, s (mph)	$0 < s \leq 20$	$20 < s \leq 30$	$30 < s \leq 40$	$40 < s \leq 50$	$50 < s \leq 60$
Frequency	30	30	40	35	5

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....

(b) 40% of the cars surveyed were fined for exceeding a certain speed as they entered the village.
Calculate an estimate of this speed. [4]

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30 + 30 + 40 + 35 + 5 = 140

40% of 140 → 14 × 4 = 56

....

36 mph

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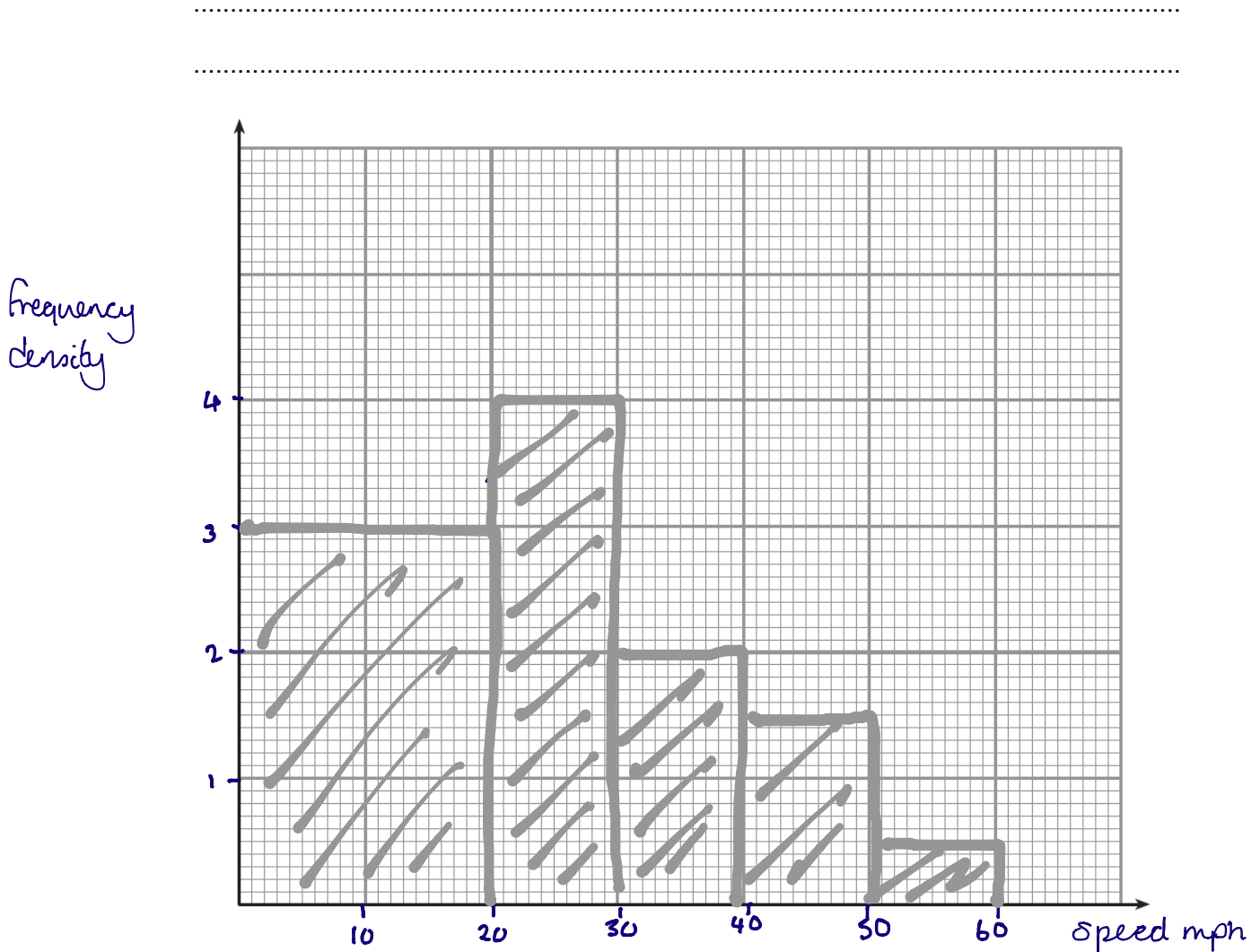
(c)   A further survey was carried out after the placement of a speed camera warning sign.

The results are summarised in the grouped frequency distribution below.

Speed, s (mph)	$0 < s \leq 20$	$20 < s \leq 30$	$30 < s \leq 40$	$40 < s \leq 50$	$50 < s \leq 60$
	FD 3 60÷20	4 40÷10	2 10	1·5 10	0·5 10
Frequency	60	40	20	15	5

Draw a histogram to illustrate the results of this survey.   [3]

....

....

(d)   Compare the two histograms. Do you consider the speed camera warning sign to have been effective?

Give a reason for your answer.   [1]

yes there are more people driving slower speeds ....

....

17. y is inversely proportional to x.

$y = 16$ when $x = \dfrac{1}{2}$.

Write an expression for y in terms of x. [4]

$$y \propto \frac{1}{x} \qquad\qquad y = \frac{k}{x}$$

when $y = 16$ $\qquad$ $16 = \dfrac{k}{0.5}$ $\qquad$ $k = 16 \times 0.5$

$x = 0.5$ $\qquad\qquad\qquad = 8$

$$y = \frac{8}{x}$$

18. Simplify $\dfrac{3\sqrt{7}}{4+\sqrt{7}}$ [4]

$$\frac{3\sqrt{7}}{4+\sqrt{7}} \times \frac{(4-\sqrt{7})}{4-\sqrt{7}} = \frac{12\sqrt{7}-3\times7}{16-4\sqrt{7}+4\sqrt{7}-7} = \frac{12\sqrt{7}-21}{9}$$

$$= \frac{4\sqrt{7}-7}{3}$$

19. Express $x^2 + 12x + 14$ in the form $(x+a)^2 + b$, where a and b are whole numbers to be found. [2]

$(x+6)^2 - 36 + 14$

$(x+6)^2 - 22$

$a = 6 \quad b = -22$

20. *(a)* Express $0.\dot{7}\dot{8}$ as a fraction. [2]

$$x = 0.787878\ldots$$
$$100x = 78.7878\ldots$$
$$99x = 78$$
$$x = \frac{78}{99} = \frac{26}{33}$$

$$\begin{array}{r} 2\cdot6 \\ 3\overline{\smash{)}78} \end{array}$$

(b) Evaluate $3^{-2} \times 9^{\frac{1}{2}}$, giving your answer as a recurring decimal. [3]

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9} \qquad\qquad 9^{\frac{1}{2}} = \sqrt{9} = 3$$

$$3 \times \frac{1}{9} = \frac{3}{9} = \frac{1}{3} = 0.\dot{3}$$

21. The diagram below shows a composite shape formed by joining two rectangles.

$48 + 12x$

$4 + x$

4 cm

$x^2 + 2x$

x cm

12 cm

$(x + 2)$ cm
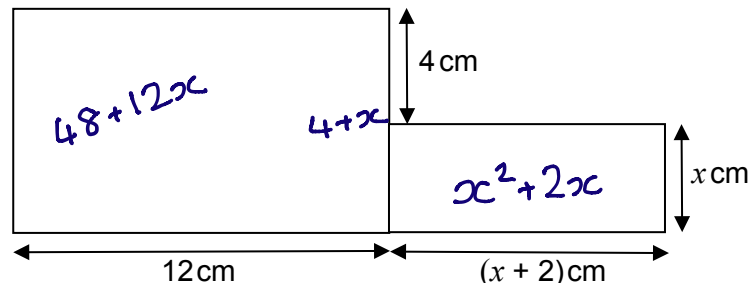
Diagram not drawn to scale

The area of the larger rectangle is 4 times the area of the smaller rectangle.

Calculate the dimensions of the smaller rectangle.
You must justify any decisions that you make. [7]

$$48 + 12x = 4(x^2 + 2x)$$
$$= 4x^2 + 8x - 12x - 48$$
$$\div 4 \quad 0 = 4x^2 - 4x - 48$$
$$x^2 - x - 12 = 0$$
$$(x - 4)(x + 3) = 0$$
$$x = 4 \quad , \quad x = -3 \text{ ← can't be a solution}$$

$$4 + 2 = 6$$
$$x = 4$$

$$\text{Dimensions} = 6 \times 4$$

22. *(a)* The diagram shows a sketch of the graph $y = f(x)$.
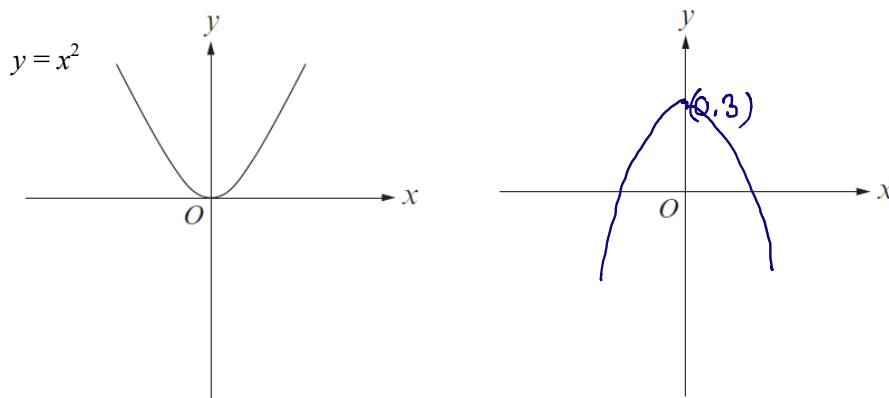The graph passes through the points $(-2, 0)$ and $(6, 0)$.

$(-2, 0)$ $(6, 0)$
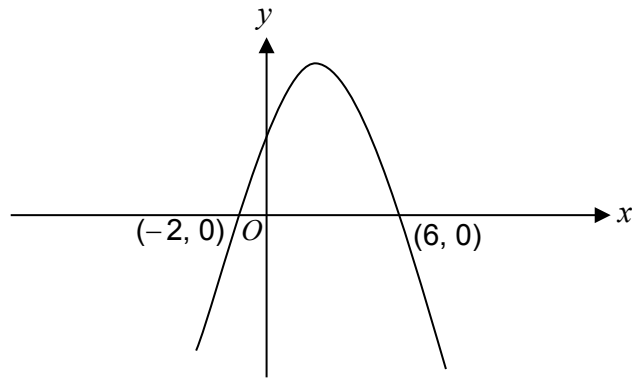
Sketch the graph of $y = f(x + 5)$ on the axes below.
You must indicate the coordinates of the points of intersection of the graph with the x-axis. [3]
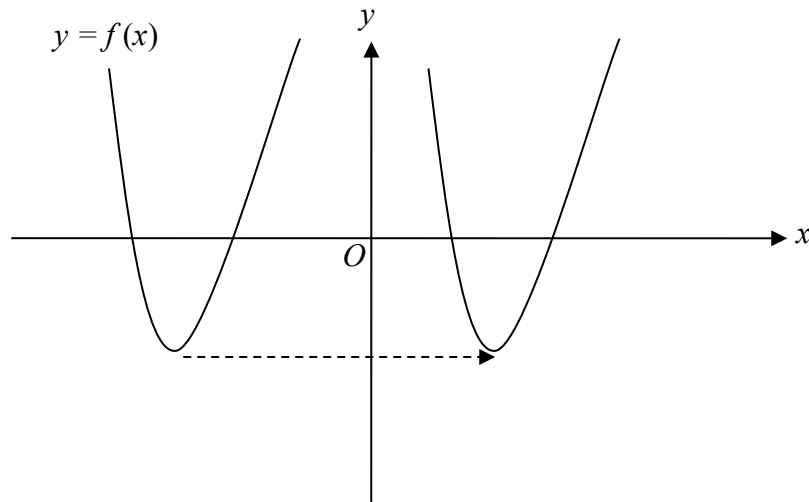
$(-7, 0)$ $(1, 0)$
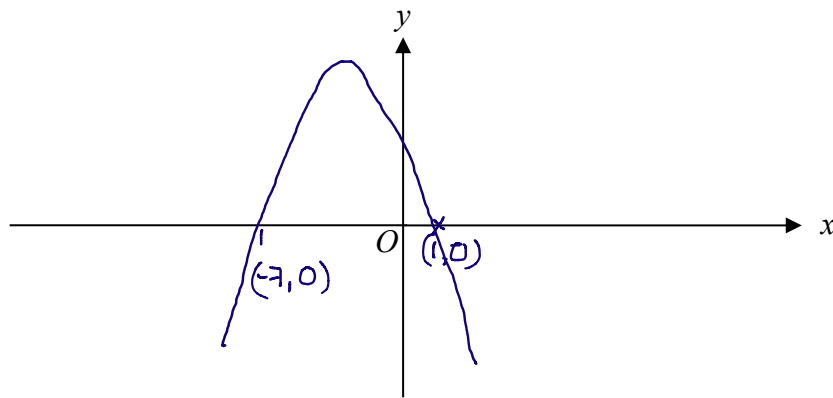
(b) The diagram below on the left shows a sketch of the graph $y = x^2$.

Sketch the graph of $y = -x^2 + 3$ on the axes on the right.
You must indicate the coordinates of the stationary point. [2]

$y = x^2$

$(0, 3)$

(c)   The function $f(x)$ has been translated, as shown in the diagram below. Explain why you cannot say exactly what translation was used.   [1]

$y = f(x)$

No co-ordinates have been given

23. *(a)* When Kayla shoots an arrow, the probability that she hits the target is $0·3$. Each attempt is independent of any previous shot.

(i) What is the probability that she hits the target for the first time on her third attempt? [2]

$P(miss) = 0.7$

$P(M,M,H) = 0.7 \times 0.7 \times 0.3$

$= 0.147$

(ii) Check whether or not there is more than a 50% chance of Kayla hitting the target **once only** on her **first three** attempts. [3]

$P(H,M,M) = 0.3 \times 0.7 \times 0.7 = 0.147$

$P(M,H,M) = 0.7 \times 0.3 \times 0.7 = 0.147$

$P(M,M,H) = 0.147$

$$\begin{array}{r} 0.147 \\ 0.147 \\ 0.147 \\ \hline 0.441 \end{array}$$

$= 0.441$ which is less than 0.5 (50%)

(b) (i) A fairground game consists of removing two balls at random from a box containing 15 blue balls and 5 red balls.

A player wins the game if two red balls are removed.

John calculates that the probability of winning the game is

$$\left(\frac{1}{4}\right)^2 = \frac{1}{16}.$$

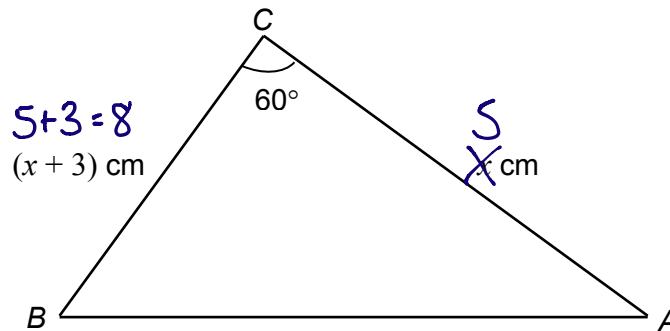What assumption has John made for his answer to be correct? [1]

$P(R,R) = \frac{5}{20} \times \frac{4}{19}$ he has assumed the 1st ball is replaced

(ii) If John's assumption was not true, what effect would this have on the probability of winning the game? [1]

the probability will be less than $\frac{1}{16}$ it will be $\frac{5}{20} \times \frac{4}{19}$

24.

C

60°

$5+3=8$

$(x+3)$ cm

5

x cm

B

A

The area of the triangle is $\sqrt{300}$ cm².

Calculate the length of *AB*.   [8]

$area = \frac{1}{2}ab\sin C$

$\sqrt{300} = \frac{1}{2} \times (x+3) \times x \times \sin 60$

$\sqrt{300} = \frac{1}{2} \times \frac{\sqrt{3}}{2} \times x^2 + 3x$

$x^2 + 3x = \frac{4\sqrt{300}}{\sqrt{3}}$

$x^2 + 3x - 40 = 0$

$(x+8)(x-5) = 0$

$x = -8 \qquad x = 5 ✓$

not a solution

1 40
2, 20
5, 8

$a^2 = b^2 + c^2 - 2bc \cos A$

$= 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 60$

$= 64 + 25 - 80 \times 0.5 \quad = 89 - 40$

$a^2 = 49$

$a = {}^{\pm}\sqrt{49} \qquad a = \pm 7$

$\therefore BA = 7 cm$