

Candidate Name	Centre Number					Candidate Number				
Just Maths						0				

**GCSE MATHEMATICS****COMPONENT 2****Calculator-Allowed Mathematics****Higher Tier****SPECIMEN PAPER****2 hours 15 minutes**

Worked Solutions

ADDITIONAL MATERIALS

A calculator will be required for this examination.

A ruler, protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided in this booklet.

Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the need for good English and orderly, clear presentation in your answers.

No certificate will be awarded to a candidate detected in any unfair practice during the examination.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	4	
2.	3	
3.	5	
4.	5	
5.	4	
6.	5	
7.	4	
8.	6	
9.	5	
10.	4	
11.	3	
12.	9	
13.	6	
14.	5	
15.	4	
16.	3	
17.	6	
18.	6	
19.	7	
20.	7	
21.	4	
22.	9	
23.	6	
TOTAL	120	

Formula list*Area and volume formulae*

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

1. (a) In an election, Stella gained 28 416 votes out of a total of 38 400 votes. [2]
Write 28 416 as a percentage of 38 400.

$$\frac{28416}{38400} \times 100 = 74\%$$

- (b) Jake needs to find a selling price which is 12% more than £766. [2]
Find the selling price.

$$766 \times 1.12 = £857.92$$

2. A survey is carried out by asking people questions as they come out of a juice bar.
A section of the questionnaire is shown below.

In questions 1 and 2 put a tick (✓) in a box

1. How old are you?

15 to 20 ☐ 21 to 30 ☐ 30 to 40 ☐ 41+ ☐

2. Do you ever go to the juice bar to buy a fruit drink?

Yes ☐ No ☐

- (a) Explain why this is a biased survey. [1]

if they are coming out of a juice bar they obviously use it.

- (b) State two criticisms of the design of question 1. [2]

First criticism of question 1:

overlapping boxes

Second criticism of question 1:

no under 15's

3. The diagram shows a square.
All the lengths are measured in centimetres.

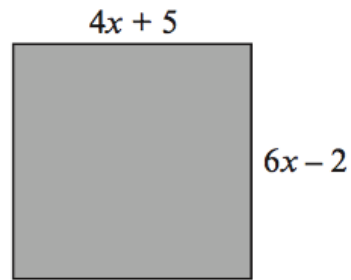


Diagram not drawn to scale

Use an algebraic method to find the length of one side of the square. [5]

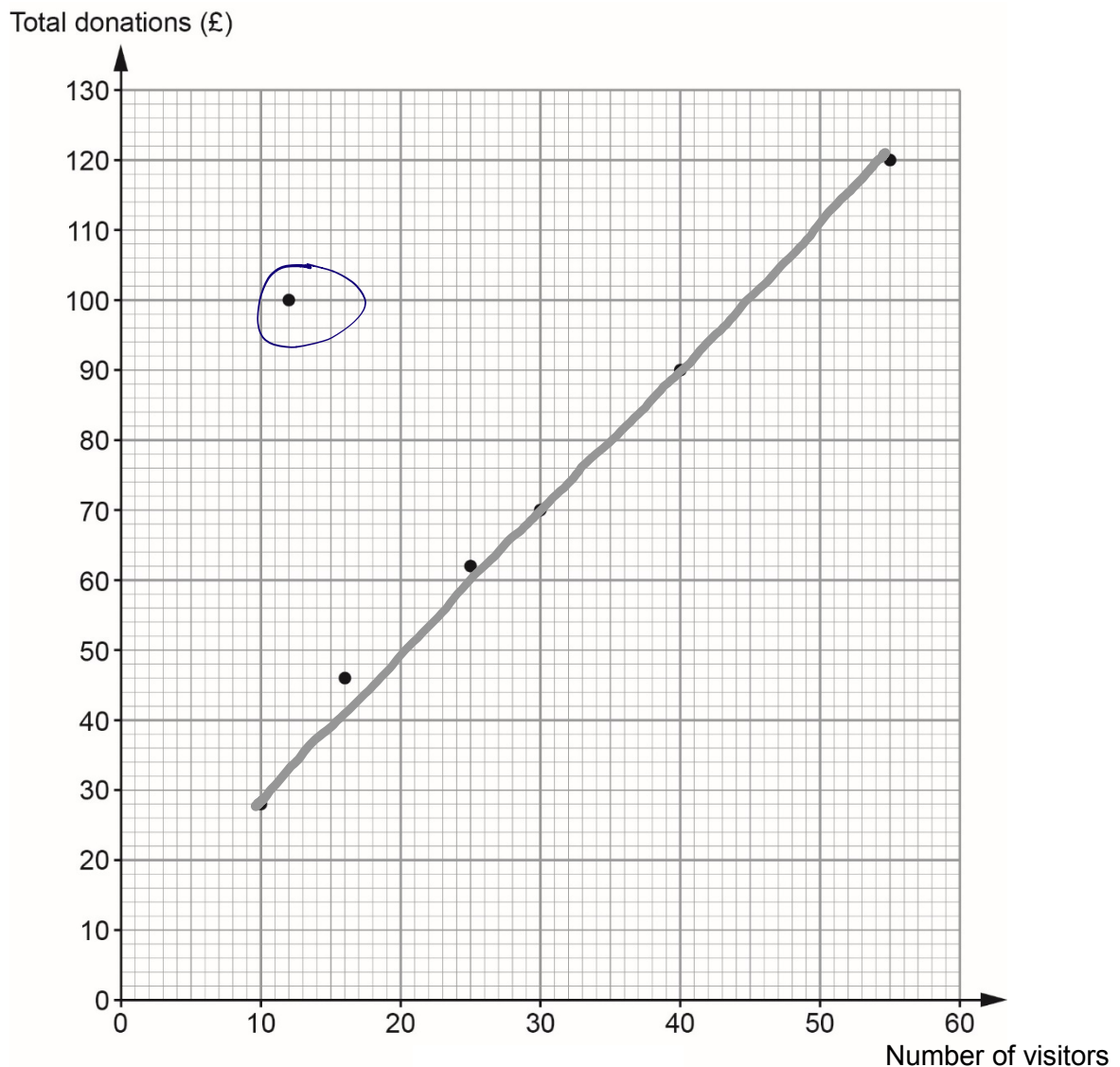
$$4x + 5 = 6x - 2$$

$$2x = 7 \quad x = \underline{3.5 \text{ cm}}$$

$$\begin{aligned} \text{so length of side} &= 4 \times 3.5 + 5 \\ &= \underline{19 \text{ cm}} \end{aligned}$$

4. The number of visitors to an animal rescue centre and the total donations received were recorded every day for 7 days.
The table and scatter diagram below show the results.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Number of visitors	40	10	16	30	25	55	12
Total donations (£)	90	28	46	70	62	120	100



- (a) Draw, by eye, a line of best fit on your scatter diagram. [1]
- (b) Describe the relationship between the number of visitors and the total donations. [1]

Is a positive correlation

- (c) Which particular day does not fit the relationship? [1]

Sunday

- (d) The animal rescue centre manager says:

"If we have 35 visitors to the centre next Wednesday we will definitely receive £80 in donations."

- (i) Explain how the manager may have come to this conclusion? [1]

he has used the line of best fit

- (ii) Is the manager's statement correct? [1]
You must give a reason for your answer.

no you can't definitely say how many donations will be received on day

5. (a) Solve $\frac{3}{x} = 12$. [1]

$$\frac{3}{12} = \frac{12x}{12} \quad x = \frac{3}{12} = \frac{1}{4}$$

(b) Solve $9x - 4 = 7(x + 2)$. [3]

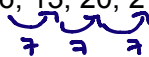
$$9x - 4 = 7x + 14$$
$$-7x \quad -7x$$

$$2x - 4 = 14$$

$$+4 \quad +4$$

$$2x = 18 \quad x = 9$$

6. (a) Find the n th term of the sequence 6, 13, 20, 27, ... [2]



$$7n - 1$$

- (b) In a sequence of four numbers, the difference between each number is 7.
The sum of the four numbers is 6.
What are the numbers in the sequence?
You must show all your working. [3]

Let first number be x

$$x, x + 7, x + 14, x + 21$$

$$\text{Sum} = 6 \quad 4x + 42 = 6$$

$$4x = 6 - 42$$

$$4x = -36$$

$$x = \frac{-36}{4} = -9$$

Numbers are $-9, -2, 5, 12$

7. A man is working out the height of a vertical tree. The man is able to measure the angle of elevation of the top of the tree from his measuring instrument. The measuring instrument is 1.8 m above ground level. When the man is standing 19 m from the base of the tree, the angle he measures is 56° .

A sketch of this situation is shown below.

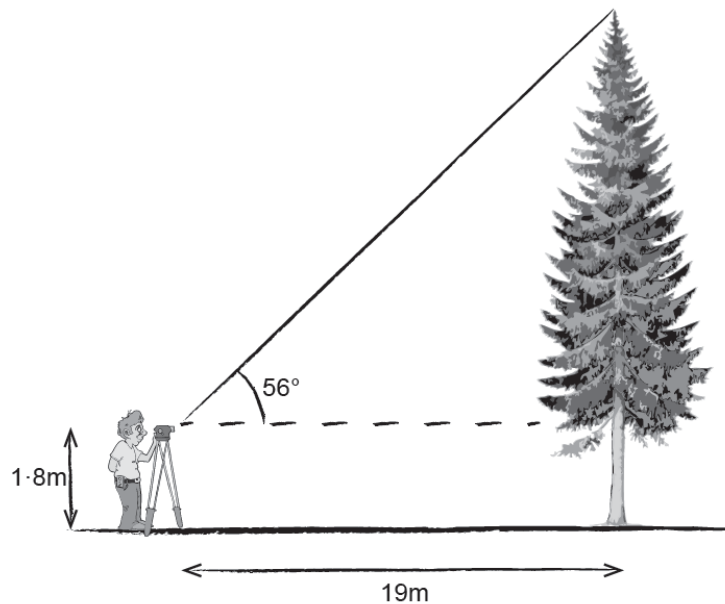
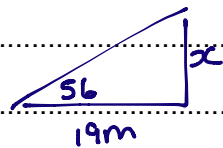


Diagram not drawn to scale

Calculate the full height of the tree.

[4]



$$\tan 56 = \frac{x}{19}$$

$$x = 19 \times \tan 56$$

$$x = 28.1686584$$

$$\text{Full height} = 28.168... + 1.8 = 29.9686584$$

$$= \underline{\underline{29.97\text{m}}}$$

8. (a) When visiting a hat shop, each customer had the circumference of their head measured.
The table shows the results for the customers who bought a hat during December.

Head circumference, c (cm)	Number of customers
$50 \leq c < 54$	12
$54 \leq c < 58$	32
$58 \leq c < 62$	14
$62 \leq c < 66$	2

Mid $f \times m$

52	624
56	1792
60	840
64	128
	<u>3384</u>

Calculate an estimate for the mean head circumference.

[4]

$3384 \div 60 = 56.4 \text{ cm}$

- (b) The hat shop sells 4 different sizes of hats.
The conversion table from head circumference to hat size is shown below.

Head circumference, c (cm)	Hat size
$50 \leq c < 54$	1
$54 \leq c < 58$	2
$58 \leq c < 62$	3
$62 \leq c < 66$	4

A salesman places an order for new stock for the hat shop.

The salesman's order form shows that about half of the hats ordered are size 2.

The owner of the shop says the order should show that about a quarter of the hats ordered are size 2.

Who is more likely to be correct, the salesman or the owner of the shop?
You must give a reason for your answer.

[2]

$\frac{32}{60} = \frac{16}{30} = \frac{8}{15}$

Salesman is correct as $\frac{32}{60}$ is closer to $\frac{1}{2}$ than $\frac{1}{4}$.

9. In an experiment, the mass added to the end of a vertical spring is gradually increased.

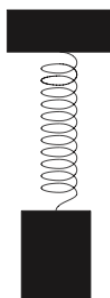
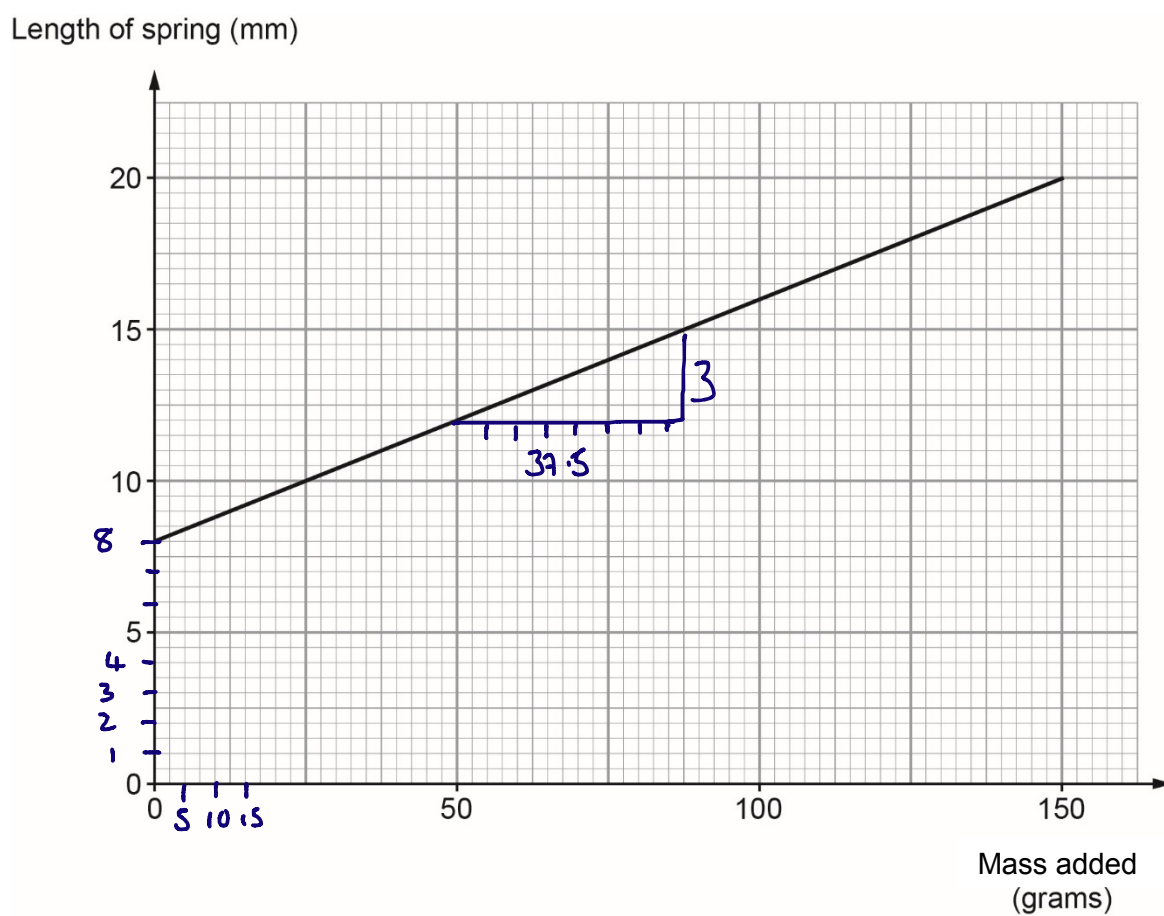


Diagram not drawn to scale

At the end of the experiment, a computer produced the graph shown below.



- (a) Write down the length of the spring without any mass added. [1]

8cm

- (b) (i) Calculate the gradient of the straight line drawn on the graph. [2]

$$\frac{3}{37.5} = 0.08$$

- (ii) Explain what the gradient of this graph tells you in relation to the experiment.

for every gram of mass the length increases by 0.08 mm [1]

- (c) The straight line stops before the right-hand edge of the graph paper.
Why do you think this might be?

[1]

- 10.** A statue is on display inside a glass cabinet.
A scale drawing of the plan view (bird's eye or aerial view) of the cuboid is shown below.



Scale 1 cm : 20 cm

A barrier is built around the cuboid so that no one can stand within 60 cm of the cuboid.

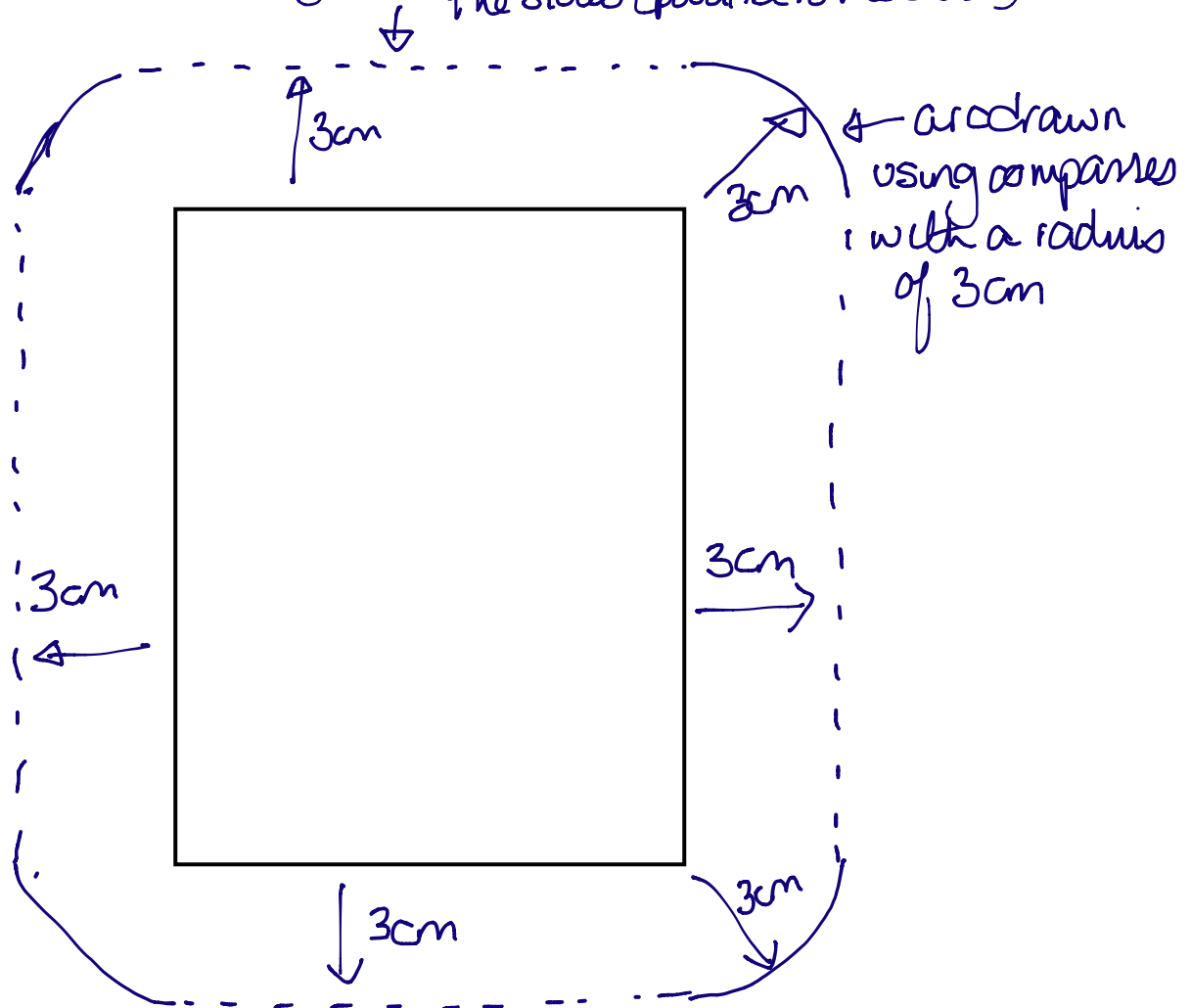
Using the given scale, draw accurately the barrier on the scale drawing shown below.

[4]

NOT DRAWN TO
SCALE ...

$\times 3 \downarrow \quad 1\text{cm} = 20\text{cm} \quad \downarrow \times 3$
 60cm

straight lines drawn 3cm away from
the sides (parallel to the sides)



11. The village of Sumston is organising a Spring Fayre to raise money for the local community centre.

(a) In the 'lucky dip', everyone wins either a toy or a pen or a pencil.
The probabilities of winning the different prizes are given in the following table.

Prize	Toy	Pen	Pencil
Probability	x	$3x$	$16x$

Find the value of x .

[2]

$$x + 3x + 16x = 20x$$

$$20x = 1$$

$$x = \frac{1}{20} = 0.05$$

- (b) In the raffle, a free second ticket is given with every ticket bought.



Stephen thinks this offer will double his chance of winning a prize.

Is Stephen correct?

You must explain your answer.

[1]

no because the total chances also doubles and so the proportion of the total remains the same

12. Laura has her own car.

During April

- Laura drove a total distance of 560 miles in her car.
- For each gallon of petrol, Laura's car travelled 37.8 miles.
- Petrol cost £1.48 per litre.
- Laura spent 10 hours 45 minutes driving her car.

$$560 \div 37.8 = 14.814 \text{ gallons}$$

- (a) 1 gallon is approximately 4.55 litres.
Calculate the cost of petrol that Laura used during April.

You must show all your working.

$$\begin{aligned} &\div 4.55 \quad 1 \text{ gallon} = 4.55 \text{ litres} \quad [5] \\ &\quad \downarrow \quad 0.219780 = 1 \end{aligned}$$

she uses 14.814 gallons

$$\div 0.219780 = 67.4037 \text{ litres}$$

$$\begin{aligned} 67.4 \times \pounds 1.48 &= 99.7629 \\ &= \pounds 99.76 \end{aligned}$$

- (b) Select which of the following best describes the roads on which Laura travelled during April.

You must show working to support your answer.

You must give a reason for your answer.

[4]

- A. Mainly small narrow country lanes
- B. Mainly inner city roads with lots of traffic lights
- C. Mainly motorways and dual carriageways
- D. Mainly roads with speed limits of 30 mph

$$10 \text{ hours } 45 \text{ mins} = 10.75 \text{ hours}$$

$$S = \frac{D}{T} \quad S = \frac{560}{10.75} = 52.0930 \text{ mph}$$

She drove mainly motorways and dual carriageways
(C)

Reason:

Her average speed was 52 mph

13. (a) During an experiment, a scientist notices that the number of bacteria **halves** every **second**.

There were 2.3×10^{30} bacteria at the start of the experiment.

Calculate how many bacteria were left after 5 seconds.

Give your answer in standard form correct to two significant figures.

[3]

$$2.3 \times 10^{30} \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5$$

$$7.1875 \times 10^{28}$$

$$= 7.2 \times 10^{28}$$

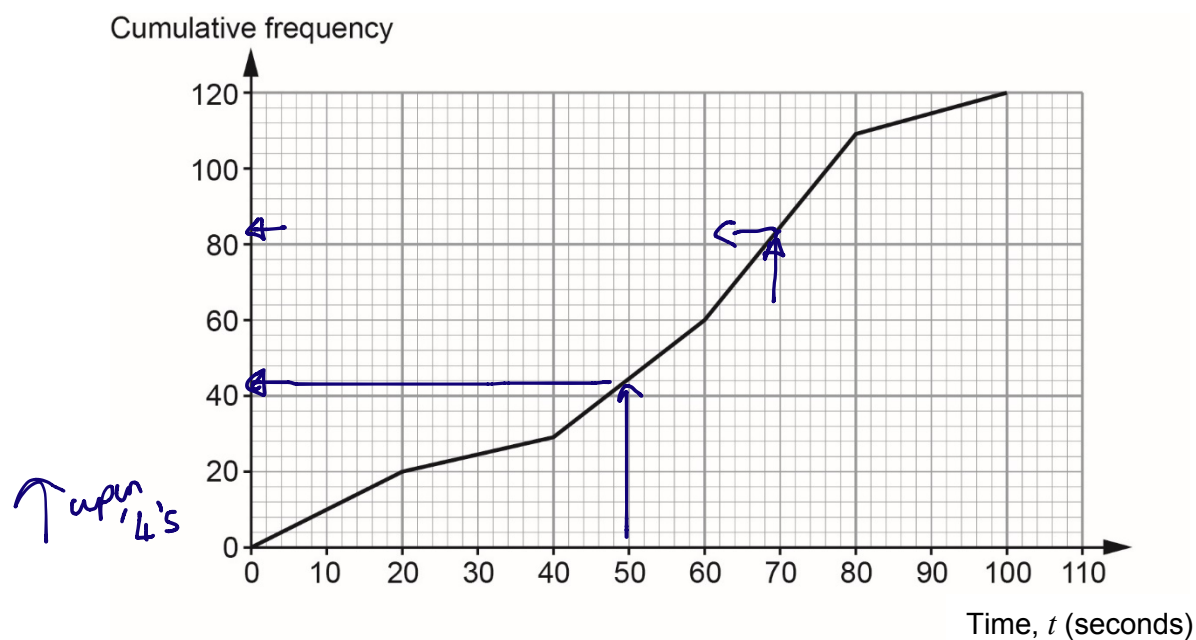
- (b) In a different experiment the number of bacteria is reduced by a quarter each second. On this occasion the number of bacteria initially was x .

Write a formula to calculate the number of bacteria, r , remaining after t seconds.

[3]

$$r = x \times 0.75^t$$

14. The times taken by customer service operators to answer 120 telephone calls are illustrated in the cumulative frequency diagram shown below.



- (a) Calculate an estimate for the percentage of telephone calls that were answered within 50 seconds.

[2]

$$\frac{44}{120} \times 100 = 36.\dot{6}\%$$

- (b) The customer service team was given a target to answer 80% of the telephone calls within 70 seconds.
Did the team meet their target?
Give a reason for your answer and state any assumption you have made when calculating your answer.
You must show all your working.

[3]

$$\text{within 70 seconds} = \frac{84}{120} \times 100 = 70\%$$

no they didnt meet the target

15.

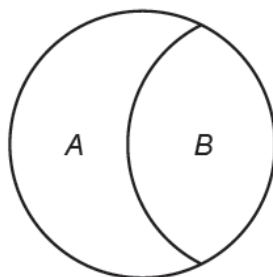


Diagram not drawn to scale

The diagram shows a circle split into two regions: A and B.

The ratio of the areas of the regions A and B is 2 : 3.
The radius of the circle is 1.5 cm.

Calculate the area of region A.

[4]

$$A = \pi \times 1.5^2 = 2.25\pi$$

$$A : B$$

$$2 : 3 = S$$

$$2.25\pi \div S = 0.45\pi$$

$$\text{Region A} = 2 \times 0.45\pi = 0.9\pi$$

$$= 2.82743388$$

$$= 2.83 \text{ cm}^2$$

16. Use the formula method to solve the equation $2x^2 + 3x - 3 = 0$.
Give your solutions correct to two decimal places.

[3]

$$a = 2 \quad b = 3 \quad c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times -3}}{2 \times 2}$$

$$= \frac{-3 \pm \sqrt{9 + 24}}{4}$$

$$= \frac{-3 \pm \sqrt{33}}{4}$$

$$\frac{-3 + \sqrt{33}}{4}$$

$$= 0.6861406616$$

$$= \underline{\underline{0.69}}$$

$$\frac{-3 - \sqrt{33}}{4}$$

$$= -2.186140662$$

$$= \underline{\underline{-2.19}}$$

17. The inside of a large industrial container has a height of 3 metres, measured correct to the nearest 10 cm.
It is used to hold a single stack of flat metal plates.
Each metal plate has a thickness of 4 centimetres, measured correct to the nearest millimetre.

- (a) Find the greatest possible number of these plates that could be stacked in the container.

[3]

$$\begin{aligned}
 & \text{LB } \nearrow 2.95 \text{ m} = 295 \text{ cm} \\
 & 3 \text{ m} \rightarrow 10 \text{ cm } 5 \text{ cm} \\
 & \text{UB } \searrow 3.05 = 305 \text{ cm} \\
 & \nearrow 3.95 \text{ cm} \qquad 305 \div 3.95 \\
 & 4 \text{ cm} \rightarrow 0.1 \cdot 0.05 \qquad = 77.2151698 \\
 & \searrow 4.05 \text{ cm} \qquad = 77 \text{ plates}
 \end{aligned}$$

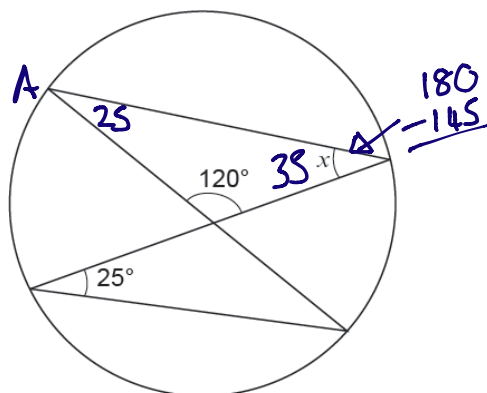
- (b) Damian states that it may not be possible to stack 73 of these plates in the container.

Show that Damian is correct.

[3]

$$\begin{array}{r}
 295 \\
 \hline
 4.05
 \end{array}
 = 72.8395 = \underline{\underline{72}}$$

18. (a)

*Diagram not drawn to scale*

Calculate the size of the angle marked x .
You must give a reason for your answer.

[2]

$$x = 35^\circ$$

$A = 25^\circ$ (angles in same segment are equal)

$$25 + 120 = 145$$

$$180 - 145 = 35 \quad \text{angles in a triangle add up to } 180^\circ$$

- (b) The diagram shows a circle with centre O .
 The tangent PT touches the circle at C .
 The reflex angle at the centre of the circle is 280° .

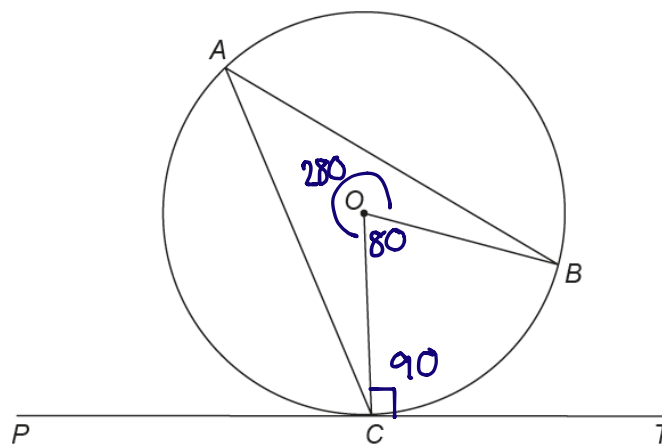


Diagram not drawn to scale

Find the size of $\hat{BAC}$.
 You must give a reason for your answer.

[2]

$$\hat{BAC} = 40$$

The angle subtended at the centre is twice the angle at the circumference

- (c) The points A , B and C lie on the circumference of a circle.
 The straight line PBT is a tangent to the circle.
 $\angle CBP = x$, where x is measured in degrees.

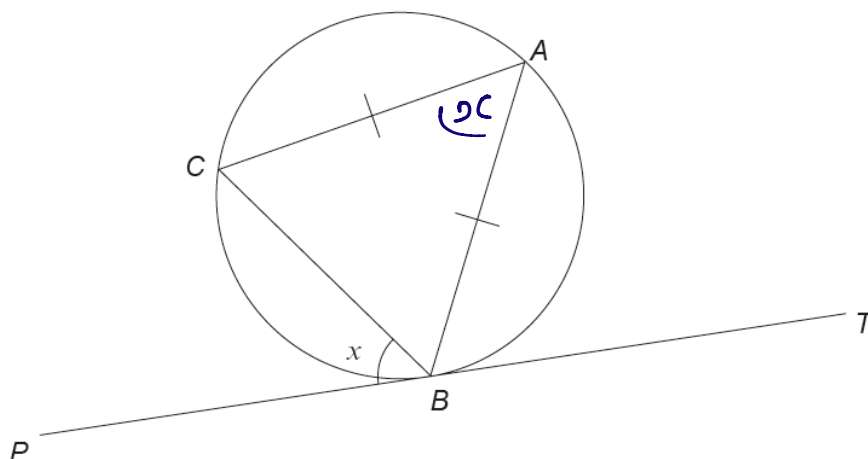


Diagram not drawn to scale

Show that the size of $\angle ABC$ in degrees is $90 - \frac{1}{2}x$.

You must give reasons for each step of your answer.

[2]

$$\angle BAC = x \quad \text{alternate segment theorem}$$

$$\angle ABC = \angle ACB = \frac{180 - x}{2} \quad \text{2 angles in an isosceles triangle are equal}$$

$$\angle ABC = \frac{180}{2} - \frac{x}{2} = 90 - \frac{1}{2}x$$

19. A cylinder is made of bendable plastic.
A dog's toy is made by bending the cylinder to form a ring.

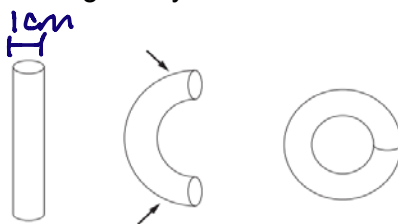


Diagram not drawn to scale

The inner radius of the dog's toy is 8 cm.
The outer radius of the dog's toy is 9 cm.

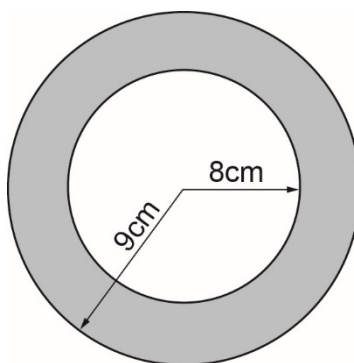


Diagram not drawn to scale

$$A = \pi r^2$$

$$C = \pi D$$

Calculate an approximate value for the volume of the dog's toy.
State and justify what assumptions you have made in your calculations and the impact they have had on your results.

$$\text{Diameter} = 1\text{cm} \quad \text{Radius} = 0.5\text{cm}$$

[7]

$$\text{Area of end of cylinder} = \pi \times 0.5^2 = \frac{1}{4}\pi$$

at least

Assumption height of cylinder is $\approx$ the circumference of circle with radius 8cm and at most circumference of circle with radius 9cm. So I'm going to use 8.5cm

$$8.5 \times 2 = 17$$

$$C = \pi \times 17 = 17\pi$$

$$\text{Volume} = \frac{\text{area of x-section}}{\times} \times \text{height} = \frac{1}{4}\pi \times 17\pi = 41.9458187$$

$$= \underline{\underline{42\text{cm}^3}}$$

20. A pebble is thrown vertically upwards.
It has an initial speed of u metres per second.
The pebble reaches a maximum height of h metres, before falling vertically downwards.
It is known that h is directly proportional to u^2 .

When the pebble is thrown with an initial speed of 10 m/s it reaches a maximum height of 5 m.

- (a) Calculate the maximum height reached when the pebble is thrown with an initial speed of 12 m/s.

[5]

$$h \propto u^2$$

$$\text{when } u = 10 \quad h = 5$$

$$h = k u^2$$

$$5 = k \times 10^2$$

$$k = \frac{5}{100} = 0.05$$

$$h = 0.05 u^2$$

$$\text{when } u = 12$$

$$h = 0.05 \times 12^2$$

$$= 7.2 \text{ m}$$

- (b) Find the initial speed of the pebble if the maximum height reached is 16 m.

[2]

$$\text{when } h = 16 \text{ m}$$

$$h = 0.05 u^2$$

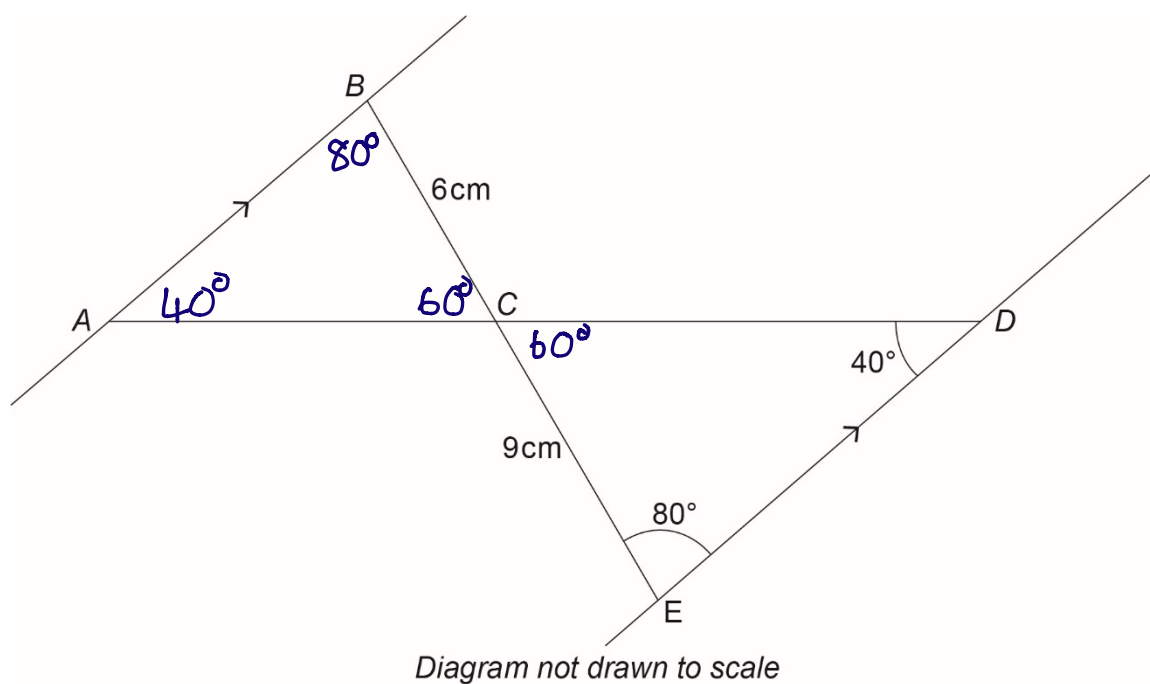
$$\frac{16}{0.05} = u^2$$

$$u = \sqrt{\frac{16}{0.05}} = \sqrt{320}$$

$$u = 17.88854382$$

$$= 17.9 \text{ m/s (1dp)}$$

21.



Given that AB is parallel to ED , calculate the length of AB .

[4]

$$\frac{6}{\sin 40} = \frac{AB}{\sin 60}$$

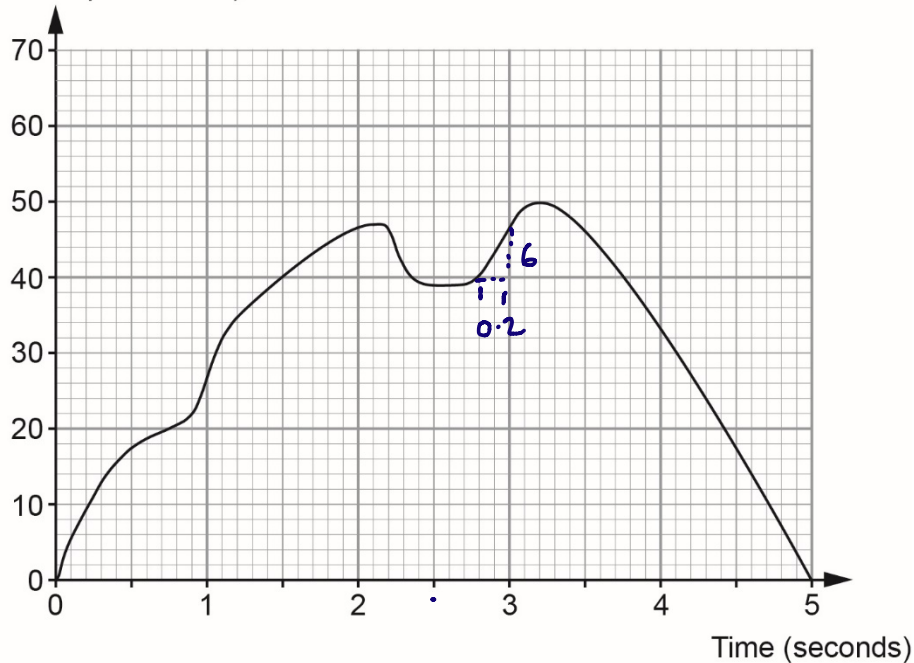
$$AB = \frac{6}{\sin 40} \times \sin 60$$

$$= 8.083778132$$

$$= 8.08 \text{ cm (2dp)}$$

22. An engineer carried out an experiment.
He recorded the velocity of a particle during the first 5 seconds of the experiment.

Velocity (metres per second)



- (a) Calculate the acceleration of the particle at 3 seconds.
You must state the units of your answer.

[4]

$$\frac{6}{0.2} = 30 \text{ ms}^{-2} \quad (\text{range of 25 to 37 is acceptable})$$

- (b) Calculate an estimate for the distance travelled by the particle in the 5 second period.

[4]

estimated area under curve
= range between 134m to 158mm

- (c) Suggest how you could improve the method you used in (b) to find a more accurate approximation of the actual distance travelled? [1]

could use smaller "trapeziums" when splitting the shape up.

23. The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = 2x$$
$$g(x) = 3 + 2x$$

- (a) Calculate the value of $gf(4)$. [2]

$$gf(x) = 3 + 2(2x) = 3 + 4x$$

$$gf(4) = 3 + 4 \times 4 = \underline{\underline{19}}$$

- (b) Solve the equation $fg(x) = 14$. [4]

$$fg(x) = 2(3 + 2x) = 6 + 4x$$

$$14 = 6 + 4x$$

$$4x = 8$$

$$x = 2$$