

Candidate Name	Centre Number	Candidate Number
Mel@JustMaths		0

SOLUTIONS



GCSE

MATHEMATICS
UNIT 1: NON-CALCULATOR
HIGHER TIER

SPECIMEN PAPER SUMMER 2017

1 HOUR 45 MINUTES

ADDITIONAL MATERIALS

The use of a calculator is not permitted in this examination.
A ruler, protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided in this booklet.

Take π as 3.14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

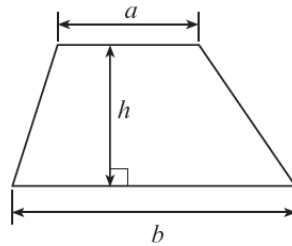
The number of marks is given in brackets at the end of each question or part-question.

The assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing in question 11.

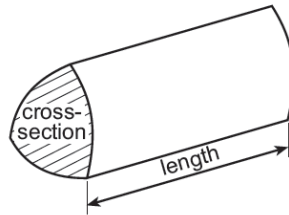
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	6	
2.	7	
3.	7	
4.	4	
5.	3	
6.	4	
7.	4	
8.	2	
9.	3	
10.	5	
11.	9	
12.	2	
13.	7	
14.	6	
15.	4	
16.	7	
TOTAL	80	

Formula list – Higher tier

Area of a trapezium = $\frac{1}{2}(a + b)h$

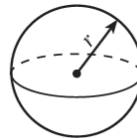


Volume of a prism = area of cross section \times length



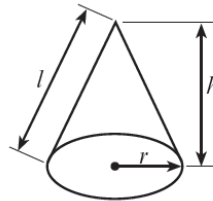
Volume of a sphere = $\frac{4}{3}\pi r^3$

Surface area of a sphere = $4\pi r^2$



Volume of a cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of a cone = $\pi r l$

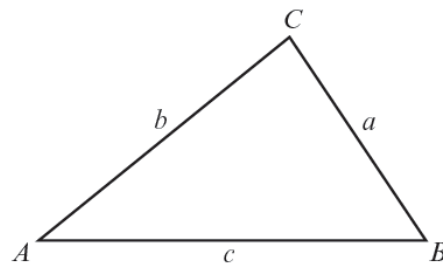


In any triangle ABC ,

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.

1. Mair either walks, cycles, travels by car or travels by bus to work each day. Her method of travel each day is independent of her method of travel on any other day.

The table below shows the probability for three of her methods of travel on any randomly chosen day.

Method of travel	Walk	Bike	Car	Bus
Probability	0.20	0.45	0.10	0.25

- (a) Calculate the probability that, on any randomly chosen day, she walks to work. [2]

$$0.45 + 0.10 + 0.25 = 0.80 \quad 1 - 0.80 \\ = 0.20$$

- (b) What is the probability that, on any randomly chosen day, she either travelled to work by car or by bus? [2]

$$0.10 + 0.25 = 0.35$$

- (c) What is the probability that, in any randomly chosen week, Mair travelled to work by car on the Monday and by bus on the Tuesday? [2]

$$0.10 \times 0.25 = 0.025$$

2. (a) The table below shows some of the values of $y = x^2 - 3x - 2$ for values of x from -2 to 4 .

Complete the table by finding the value of y for $x = 2$.

[1]

x	-2	-1	0	1	2	3	4
$y = x^2 - 3x - 2$	8	2	-2	-4	-4	-2	2

.....
 $y = 2^2 - 3 \times 2 - 2 = 4 - 6 - 2 =$

- (b) On the graph paper opposite, draw the graph of $y = x^2 - 3x - 2$ for values of x from -2 to 4 . [2]

- (c) Using your graph, write down the two solutions of the equation $x^2 - 3x - 2 = 0$. Give your answers correct to 1 decimal place. [1]

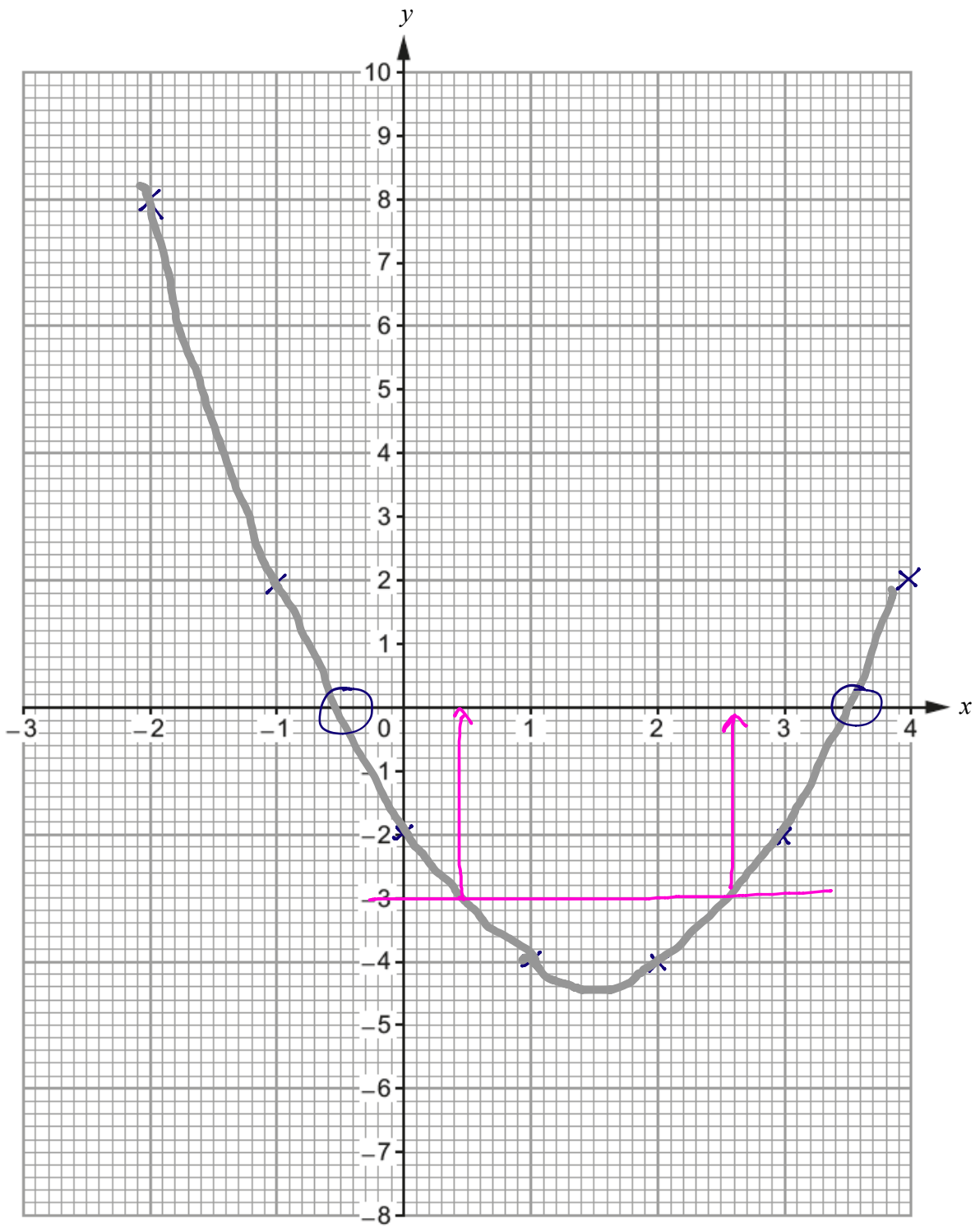
Solutions are -0.5 and 3.5

- (d) By drawing a suitable line on your graph, write down the two solutions of the equation $x^2 - 3x + 1 = 0$. Give your answers correct to 1 decimal place. [3]

.....

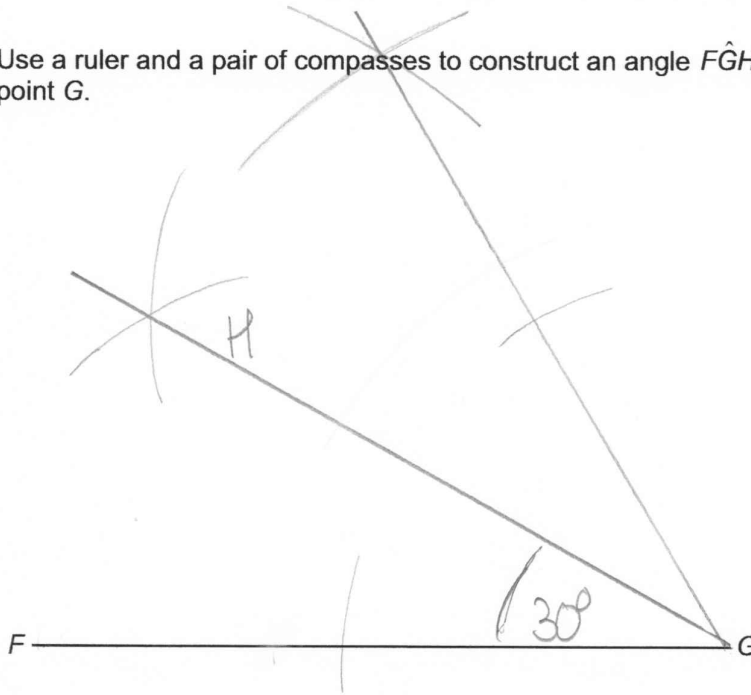
Solutions are 0.4 and 2.6

For use with question 2.



3

- (a) Use a ruler and a pair of compasses to construct an angle $F\hat{G}H$ of size 30° at point G. [3]

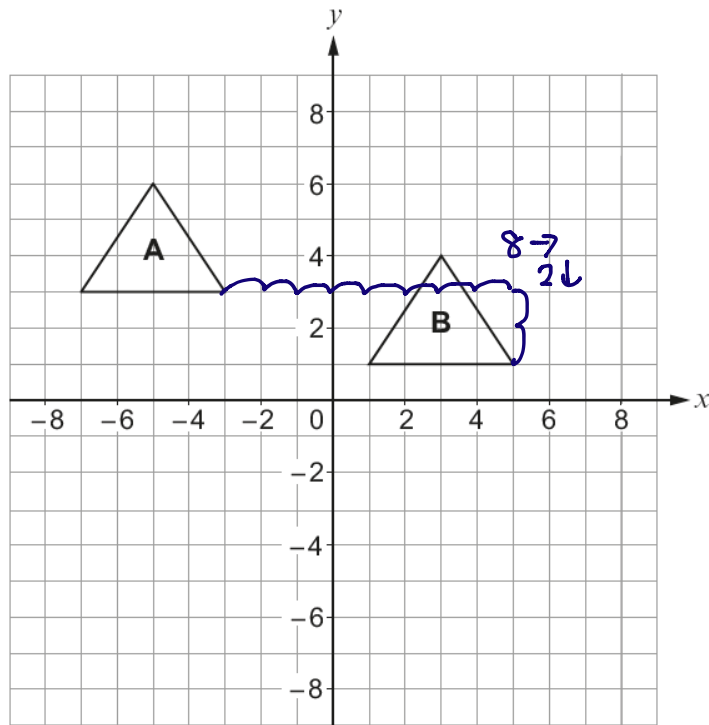


- (b) A regular polygon has interior angles of 135° . How many sides does this polygon have? [3]

exterior angle = $180 - 135 = 45$

$360 \div 45 = 8$ sides

- (c) Shape A is translated onto Shape B.



Which one of the following vectors describes the translation?
Circle your answer.

[1]

$\begin{pmatrix} 8 \\ -2 \end{pmatrix}$

$\begin{pmatrix} 2 \\ -8 \end{pmatrix}$

$\begin{pmatrix} -8 \\ -2 \end{pmatrix}$

$\begin{pmatrix} -2 \\ 8 \end{pmatrix}$

$\begin{pmatrix} -8 \\ 2 \end{pmatrix}$

4. (a) Calculate the largest share when £400 is shared in the ratio 1:2:5, [2]

$400 \div 8 = 50$ $50 \times 5 = \underline{\underline{£250}}$

- (b) A price of £63 includes VAT at a rate of 5%.
What was the price before VAT was added? [2]

$\begin{matrix} \div 1.05 & 63 = 105\% & \div 21 \\ \swarrow & \searrow & \\ \text{£}3 & 5\% & \\ \text{£}60 & 100\% & \times 20 \end{matrix} = \underline{\underline{£60}}$

5. Circle your answer in each of the following.

$$\frac{1}{2^3} = \frac{1}{8}$$

(a) The value of 2^{-3} as a fraction in its simplest form is

$\frac{1}{6}$

$-\frac{1}{6}$

$-\frac{1}{8}$

$\frac{1}{8}$

$-\frac{2}{3}$

[1]

(b) $\frac{2}{9}$ as a recurring decimal is

$\frac{1}{9} = 0.1111\dots$

$\frac{2}{9} = 0.222\dots$

0.2929.....

0.2999.....

0.9292.....

0.9222.....

0.2222.....

[1]

(c) 17^0 is equal to

17

1

0

$\frac{1}{17}$

1.7

[1]

6. A six-sided dice was thrown repeatedly.
After every 100 throws, the **cumulative** number of sixes thrown was recorded.

(a) Complete the table below, which gives a summary of the results obtained.

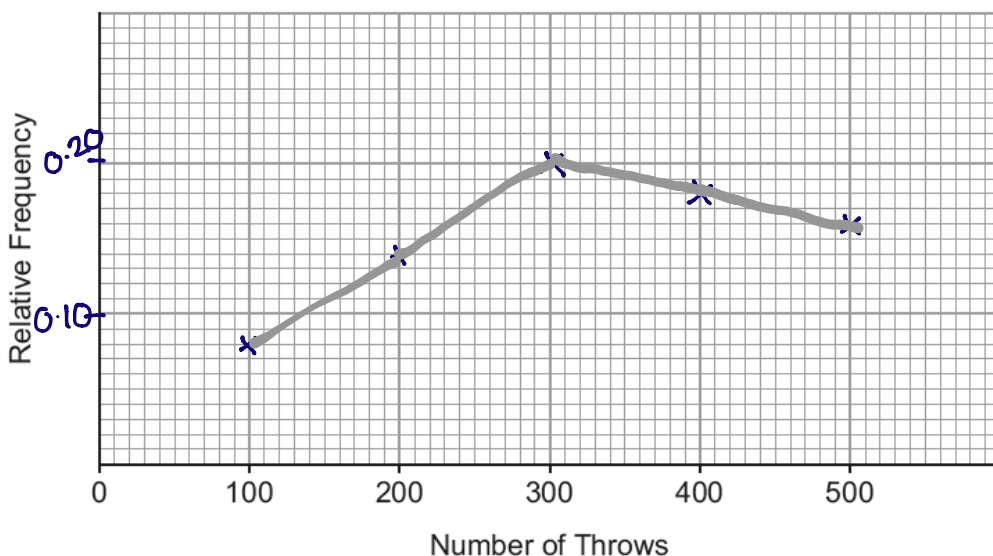
[1]

Number of throws	100	200	300	400	500
Number of sixes	8	28	60	72	80
Relative frequency	0.08	0.14	0.20	0.18	0.16

$$\frac{60}{300} = \frac{1}{5} = 0.2 \quad \frac{80}{500} = \frac{16}{100} = 0.16$$

(b) Draw a relative frequency diagram to show the information given in the table.

[1]



(c) From the table, which value gives the best estimate for the probability of throwing a six? You must give a reason for your choice.

[1]

0.16 because it comes from the greatest number of throws

(d) Do you think this is a fair dice? You must give a reason for your choice.

[1]

$\frac{1}{6} = 0.1666$ yes because $\frac{1}{6}$ is close to 0.16

7. Find, in standard form, the value of

(a) $(4.1 \times 10^{-5}) \times 3000$, [2]

$$4.1 \times 10^{-5} \times 3 \times 10^3 = 12.3 \times 10^{-2}$$

$$= 0.123 = 1.23 \times 10^{-1}$$

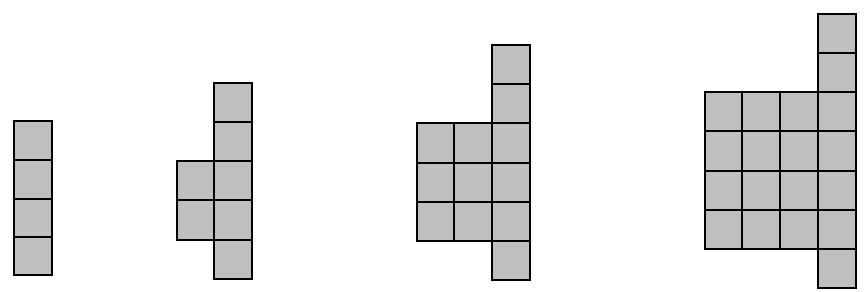
(b) $(1.5 \times 10^3) \div (3 \times 10^6)$. [2]

$$\frac{1.5 \times 10^3}{3 \times 10^6} = 0.5 \times 10^{-3}$$

$$= 0.0005$$

$$= 5 \times 10^{-4}$$

8. The diagram shows the first four patterns of a sequence.



1 2 3 4
 4 7 12 19

Find an expression for the number of squares in the n th pattern of the sequence. [2]

n	1	2	3	4	
	4	7	12	19	
		3	5	7	
		2	2		

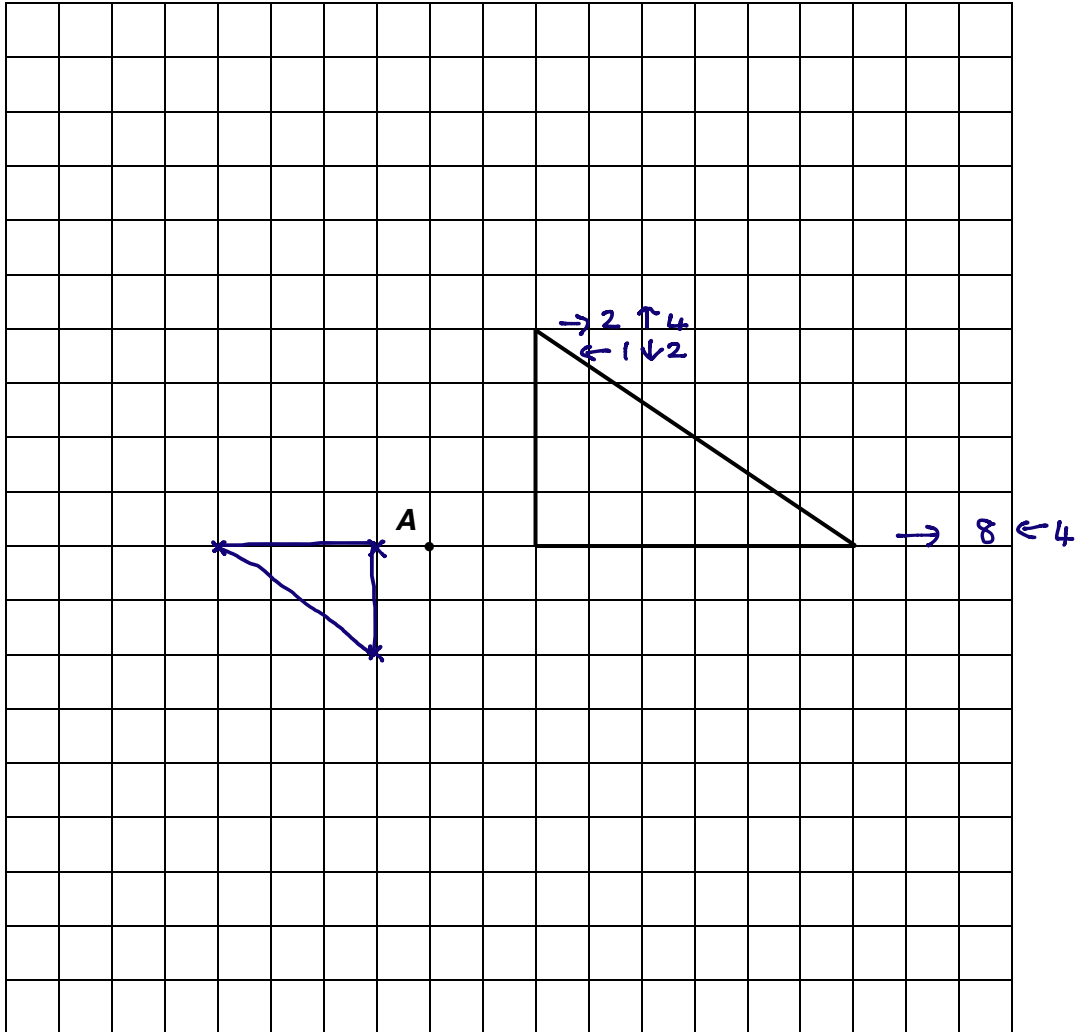
1st. difference is not the same so its quadratic.

n^2	1	4	9	16	
	+3	+3	+3	+3	

$n^2 + 3$

9. On the grid below, draw an enlargement of the given shape using a scale factor of $-\frac{1}{2}$ and centre **A**.

[3]



10. Given that y is inversely proportional to x^2 , and that $y = 5$ when $x = 2$,

(a) find an expression for y in terms of x . [3]

$$y \propto \frac{1}{x^2} \qquad y = \frac{k}{x^2}$$

$$\text{when } y = 5 \text{ and } x = 2 \qquad 5 = \frac{k}{2^2} \qquad k = 5 \times 4$$

$$y = \frac{20}{x^2}$$

(b) Use the expression you found in (a) to complete the following table. [2]

x	2	0.5	10
y	5	80	0.2

$$x = 0.5 \quad y = \frac{20}{0.5^2} = 20 \div \frac{1}{4} = 80$$

$$y = 0.2 = \frac{1}{5} \quad \frac{1}{5} = \frac{20}{x^2} \quad x^2 = 20 \div \frac{1}{5} = 100$$

$$x = \sqrt{100} = 10$$

11. You will be assessed on the quality of your organisation, communication and accuracy in writing in this question.

A cuboid with a volume of 912 cm^3 has dimensions 4 cm, $(x + 2)$ cm and $(x + 9)$ cm.

✓ Show that $x^2 + 11x - 210 = 0$.

Solve this equation and find the dimensions of the cuboid.
You must justify any decisions that you make.

[9]

$$4 \times (x+2) \times (x+9) = 912$$

$$(4x+8)(x+9)$$

$$4x^2 + 36x + 8x + 72 - 912 = 0$$

$$4x^2 + 44x - 840 = 0$$

$$\begin{array}{r} 8 \overline{)912} \\ - 72 \\ \hline 840 \end{array}$$

$$\div 4$$

$$x^2 + 11x - 210 = 0 \quad \text{GED}$$

$21 \times 10?$

$$(x+21)(x-10) = 0$$

$$x = -21 \quad \text{or} \quad x = 10$$

not a solution

Dimensions are $10+2 = 12 \text{ cm}$

$$10+9 = 19 \text{ cm}$$

4 cm

12. Circle your answer in each of the following.

(a) $(2a^3)^4$ is equal to

$2a^{12}$

$8a^{12}$

$16a^7$

$16a^{12}$

$24a^{34}$

[1]

(b) Given that $h^2 = a^2 + b^2$, then b is equal to

$h - a$

$\pm \sqrt{h^2 - a^2}$

$h^4 - a^4$

$\frac{(h^2 - a^2)}{2}$

$\frac{\pm \sqrt{h^2 - a^2}}{2}$

[1]

$2^4 a^{12} \quad 16 a^{12}$

$\sqrt{h^2 - a^2}$

13. (a) Express $0.\dot{4}\dot{7}\dot{8}$ as a fraction. [2]

$$\begin{aligned}
 x &= 0.\dot{4}\dot{7}\dot{8} \\
 100x &= 47.\dot{8}\dot{7}\dot{8} \\
 \hline
 99x &= 47.4 \\
 x &= \frac{47.4}{99} = \frac{474}{990}
 \end{aligned}$$

- (b) Find the values of a and b , given that $(4 - \sqrt{3})^2 = a + b\sqrt{3}$. [3]

$$\begin{aligned}
 (4 - \sqrt{3})(4 - \sqrt{3}) \\
 16 - 4\sqrt{3} - 4\sqrt{3} + 9 = 25 - 8\sqrt{3}
 \end{aligned}$$

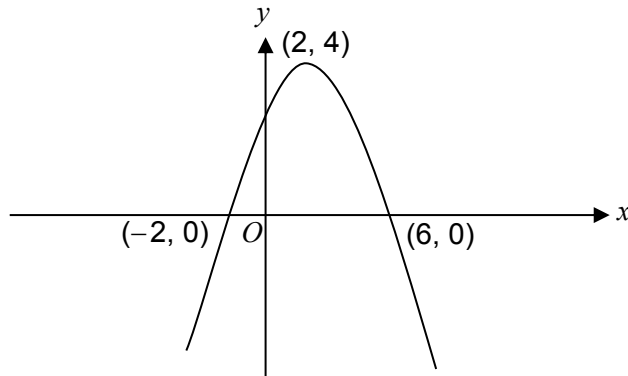
$$a = 25 \qquad b = -8$$

- (c) Evaluate $27^{-\frac{2}{3}}$. [2]

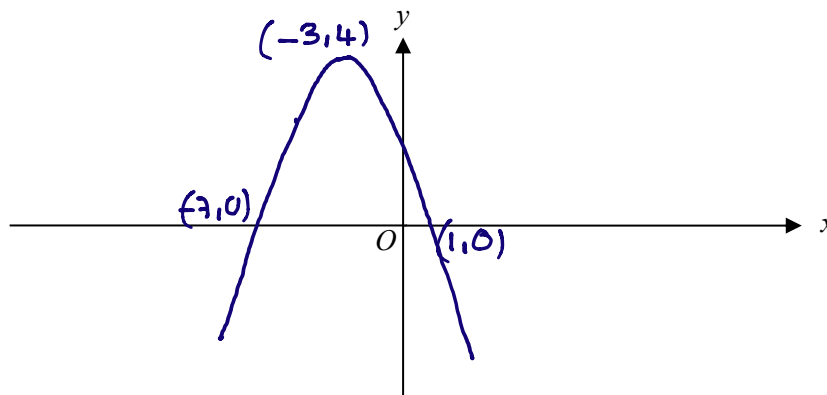
$$\frac{1}{27^{\frac{2}{3}}} = \left(\frac{1}{\sqrt[3]{27}}\right)^2$$

$$= \frac{1}{3^2} = \frac{1}{9}$$

14. (a) The diagram shows a sketch of the graph $y = f(x)$.
The graph passes through the points $(-2, 0)$ and $(6, 0)$ and its highest point is at $(2, 4)$.

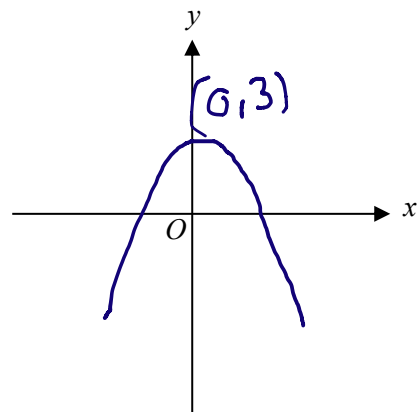
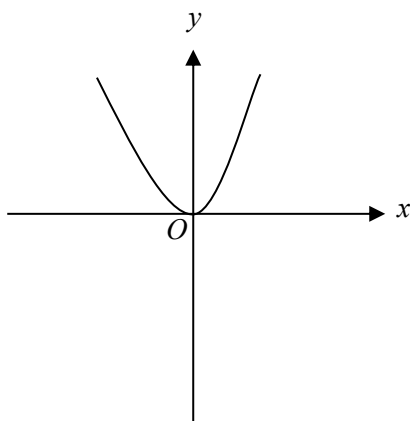


- Sketch the graph of $y = f(x + 5)$ on the axes below.
You must indicate the coordinates of its highest point and the coordinates of the points of intersection of the graph with the x -axis. [3]

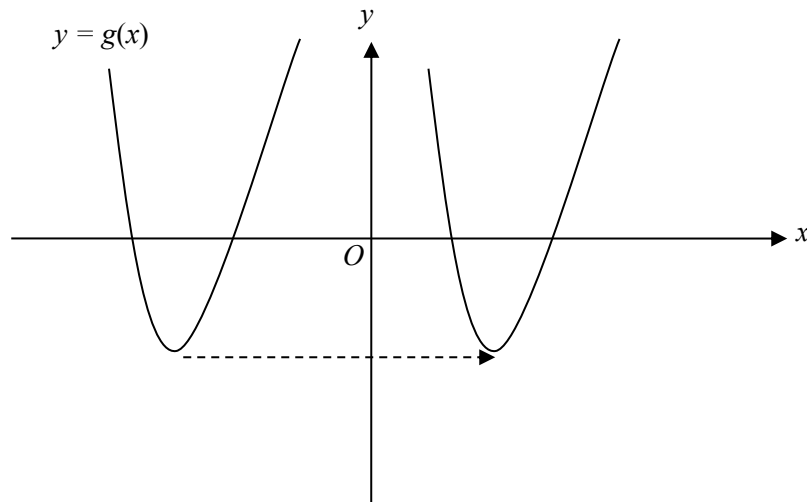


- (b) The diagram below on the left shows a sketch of the graph of $y = x^2$.

- Sketch the graph of $y = -x^2 + 3$ on the axes on the right.
You must indicate the coordinates of the point where the curve crosses the y -axis. [2]



- (c) Explain why it is not possible to determine the translation used on the function $g(x)$ in the diagram below. [1]



no scale is shown.

15. The points A , B and C lie on the circumference of a circle.

The straight line PBT is a tangent to the circle.

$AB = AC$.

$\hat{CBP} = x$, where x is measured in degrees.

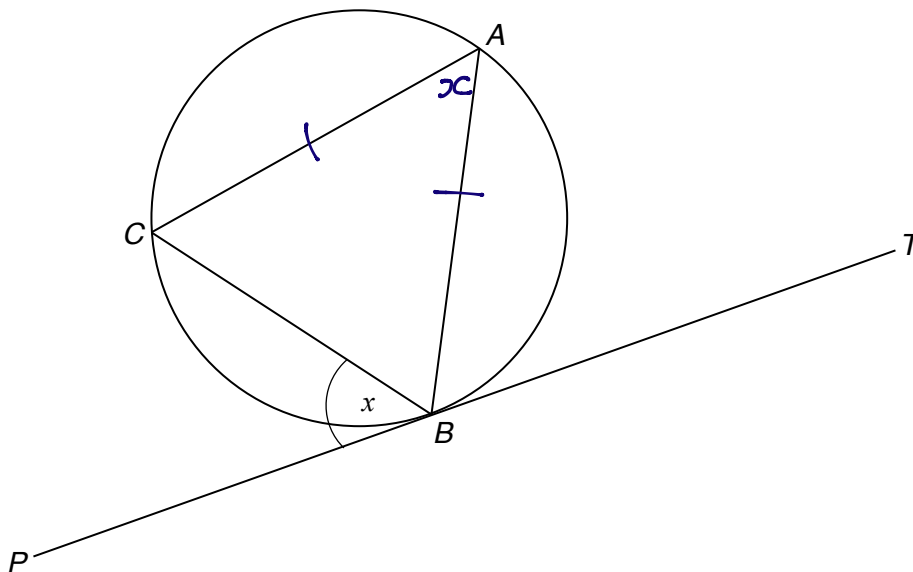


Diagram not drawn to scale

Show, giving reasons in your answer, that the size of \hat{ABC} , in degrees, is $90 - \frac{1}{2}x$.

[4]

$$CAB = x \text{ alternate segment theory}$$

$$AB = AC \text{ so } ACB = ABC = \frac{180 - x}{2}$$

$$= 90 - \frac{x}{2} \text{ QED}$$

16. (a) When Anna shoots an arrow, the probability that she hits the target is 0.3. Each attempt is independent of any previous shot.

- (i) What is the probability that Anna hits the target for the first time on her third attempt? [3]

$$m \ m \ H$$

$$0.7 \times 0.7 \times 0.3 = 0.147$$

$$7 \times 7 = 49$$

$$\begin{array}{r} \times 3 \\ \hline 147 \\ \hline \end{array}$$

H H H
 H H m
 H m H
 H m m
 m H H
 m H m
 m m H
 m m m

- (ii) Evaluate whether or not there is more than a 50% chance of Anna hitting the target **exactly once** on her **first three** attempts. [3]

$$H \ m \ m = 0.3 \times 0.7 \times 0.7 \quad 0.147$$

$$m \ m \ H = 0.7 \times 0.7 \times 0.3 \quad 0.147$$

$$m \ H \ m = 0.7 \times 0.3 \times 0.7 \quad 0.147$$

$$\hline 0.441$$

no 0.441 is less than 50%.

- (b) Siôn selects two balls, at random, from a box containing 15 blue balls and 5 red balls.

He calculates that the probability of selecting two red balls is

$$\left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

- What assumption has Siôn made for his answer to be correct? [1]

The first ball is replaced