

Candidate Name	Centre Number					Candidate Number				
Mel@JustMaths						0				

SOLUTIONS



GCSE

MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

SPECIMEN PAPER SUMMER 2017

1 HOUR 45 MINUTES

ADDITIONAL MATERIALS

A calculator will be required for this paper.

A ruler, protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided in this booklet.

Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

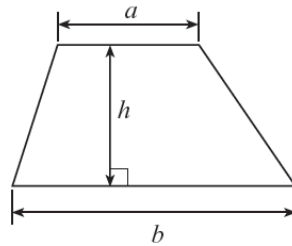
The number of marks is given in brackets at the end of each question or part-question.

The assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing in question 9(b).

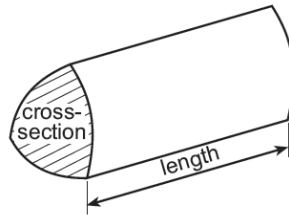
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	5	
2.	5	
3.	3	
4.	4	
5.	4	
6.	6	
7.	3	
8.	3	
9.	8	
10.	5	
11.	4	
12.	3	
13.	7	
14.	4	
15.	3	
16.	2	
17.	6	
18.	5	
TOTAL	80	

Formula list – Higher tier

Area of a trapezium = $\frac{1}{2}(a + b)h$

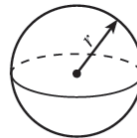


Volume of a prism = area of cross section \times length



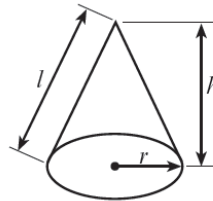
Volume of a sphere = $\frac{4}{3}\pi r^3$

Surface area of a sphere = $4\pi r^2$



Volume of a cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of a cone = $\pi r l$

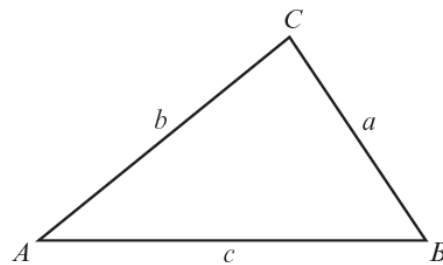


In any triangle ABC ,

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.

1. Four of the interior angles of a seven-sided polygon are 114° , 150° , 160° and 170° . The other three interior angles of this polygon are equal. Calculate the size of each of the other three interior angles. [5]

$$3 \text{ sides} = 180$$

$$4 = 360$$

$$5 = 540$$

$$6 = 720$$

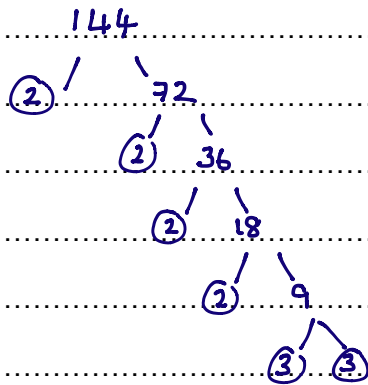
$$7 = 900^\circ$$

$$114 + 150 + 160 + 170 = 594$$

$$900 - 594 = 306$$

$$306 \div 3 = \underline{\underline{102^\circ}}$$

2. (a) Express 144 as the product of its prime factors in index form. [3]



$$2^4 \times 3^2$$

- (b) Given that $60 = 2^2 \times 3 \times 5$, find

- (i) the highest common factor (HCF) of 144 and 60, [1]

$$60 = 2 \times 2 \times 3 \times 5$$

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$\text{HCF} = 2 \times 2 \times 3 = \underline{\underline{12}}$$

- (ii) the lowest common multiple (LCM) of 144 and 60. [1]

$$\text{LCM} = 12 \times 2 \times 2 \times 5 \times 3$$

$$= 12 \times 60 = \underline{\underline{720}}$$

3. (a) Solve the inequality given below. [2]

$$7n < 5n + 11$$

$$2n < 11$$

$$n < 5.5$$

- (b) Give the largest integer value for n that satisfies this inequality. [1]

$$n = 5$$

4. A solution to the equation

$$x^3 - 7x - 75 = 0$$

lies between 4 and 5.

Use the method of trial and improvement to find this solution correct to 1 decimal place.

You must show all your working.

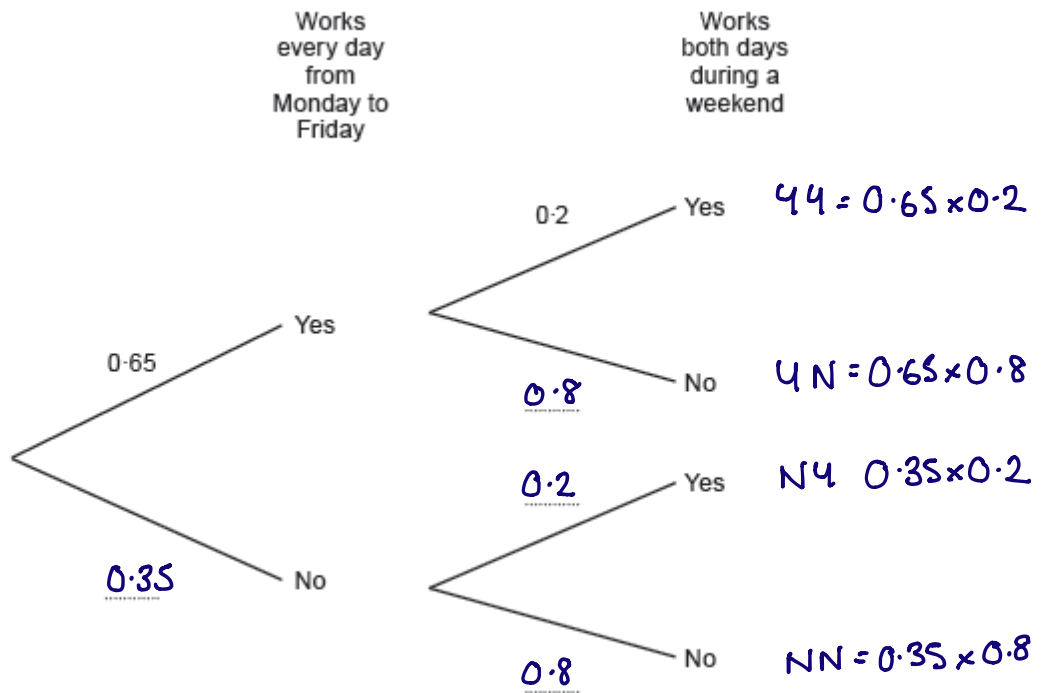
[4]

x	$x^3 - 7x - 75$	Comment
4.5	-15.375	too small
4.8	1.992	too big
4.7	-4.077	too small
4.75	-1.078125	too small (choose larger)

$$\underline{\underline{x = 4.8}}$$

5. Carys has a Monday to Friday job and a weekend job.
 Working Monday to Friday and working weekends are independent events.
 In any given week, the probability that Carys works every day from Monday to Friday is 0.65.
 The probability that she works both days during a weekend is 0.2.

(a) Complete the following tree diagram. [2]



(b) Calculate the probability that next week Carys will work every day from Monday to Sunday. [2]

$P(Y,Y) = 0.65 \times 0.2 = 0.13$

.....

.....

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6. An allotment has two rectangular flower beds A and B.

Flower bed A is x metres long and y metres wide.

Flower bed B is twice as long as flower bed A and is 3 metres wider than flower bed A.

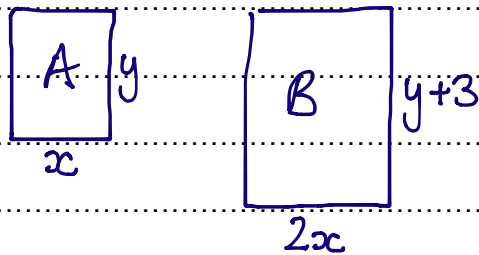
The perimeter of flower bed A is 18 metres.

The perimeter of flower bed B is 34 metres.

Use an algebraic method to calculate the area of flower bed B.

You must show all your working.

[6]



$$\text{Perimeter of A} = 2x + 2y = 18$$

$$2x = 18 - 2y$$

$$\text{Perimeter of B} = 4x + 2y + 6 = 34$$

$$\text{so } 4x = 36 - 4y$$

$$36 - 4y + 2y + 6 = 34$$

$$-2y = 34 - 42$$

$$-2y = -8 \quad y = 4$$

$$2x = 18 - 8 = 10$$

$$x = 5$$

$$\text{Area of B} = (2 \times 5) \times (4 + 3) = 10 \times 7 = \underline{\underline{70 \text{ m}^2}}$$

7. Factorise $x^2 - x - 20$, and hence solve $x^2 - x - 20 = 0$. [3]

$$(x-5)(x+4) = 0$$

$$x = 5 \text{ or } x = -4$$

8. A sketch of the graph of the straight line $y = 7x + 2$ is shown below.

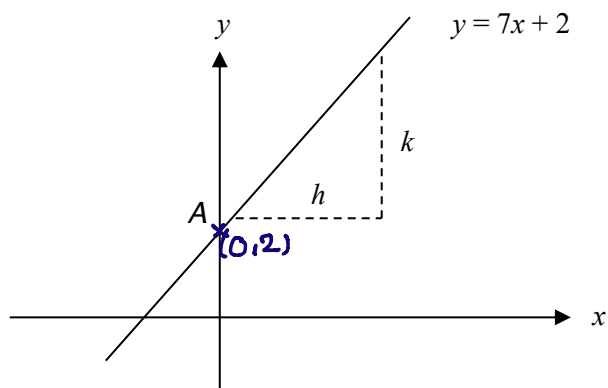


Diagram not drawn to scale

- (a) What are the coordinates of the point A, where the line cuts the y -axis?
Circle your answer. $x = 0 \quad y = 2$ [1]

(2, 0) (7, 0) (0, 2) (0, 7) (7, 2)

- (b) When h is equal to 1 unit, what is the value of k ?
Circle your answer. [1]

2 units 7 units 1 unit 3.5 units 14 units

- (c) Which of the following equations is an equation of a straight line that is
perpendicular to $y = 7x + 2$?
Circle your answer. $\text{gradient} = 7$
 $\text{perpendicular} = -\frac{1}{7}$ [1]

$y = 7x + 3$ $y = \frac{x}{7} + 3$ $y = 7x + 3$ $y = -\frac{x}{7} + 3$ $y = 2x + 7$

9.

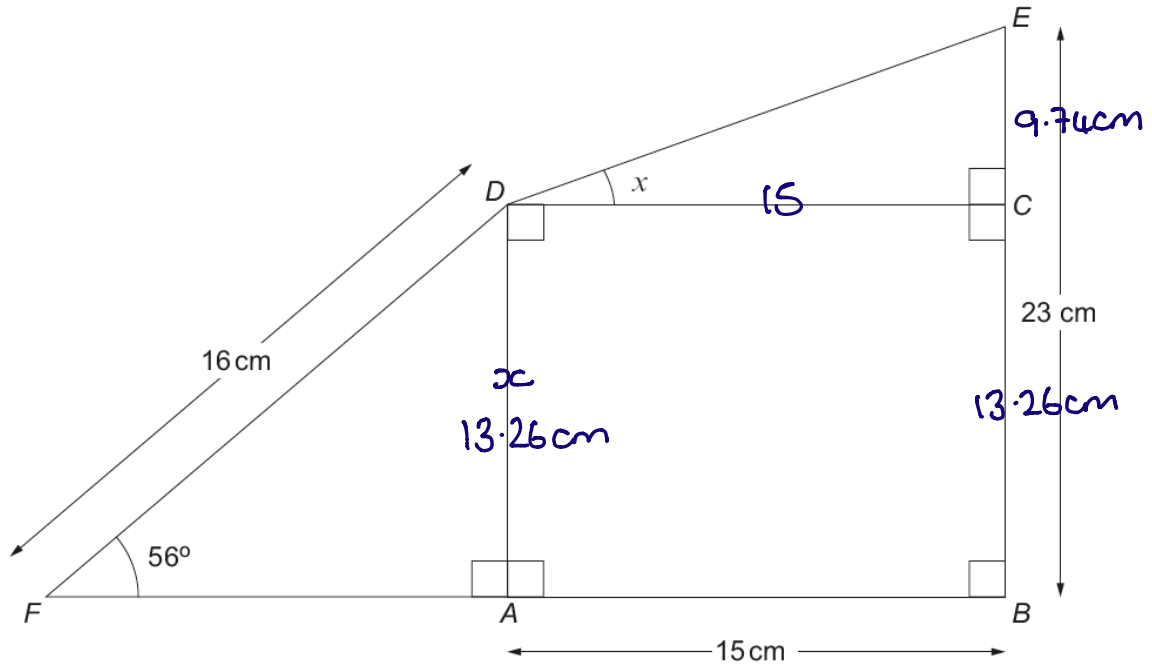


Diagram not drawn to scale

(a) Calculate the length AD .

[3]

$$\sin 56 = \frac{x}{16}$$

$$x = \sin 56 \times 16 = 13.26460116$$

$$= 13.26 \text{ cm}$$

- (b) You will be assessed on the quality of your organisation, communication and accuracy in writing in this part of the question

Find the size of the angle x .

[5]

$$CE = 23 - 13.264\dots = 9.735398839 = 9.74 \text{ (2dp)}$$

$$\tan x = \frac{9.74\dots}{15}$$

$$x = \tan^{-1}\left(\frac{9.74\dots}{15}\right)$$

$$= 32.98464286 = 32.98^\circ \text{ (2dp)}$$

10. (a) Make c the subject of the following formula.

[2]

$$\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$$

$$\times abc \quad \frac{abc}{a} - \frac{abc}{b} - \frac{abc}{c} = 0$$

$$bc - ac = ab$$

$$c(b-a) = ab$$

$$c = \frac{ab}{(b-a)}$$

- (b) Solve $3x^2 + 4x - 18 = 0$, giving your answers correct to two decimal places. You must show all your working.

[3]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 3 \quad b = 4 \quad c = -18$$

$$x = \frac{-4 \pm \sqrt{16 - 4 \times 3 \times -18}}{6}$$

$$= \frac{-4 \pm \sqrt{232}}{6}$$

$$\frac{-4 + \sqrt{232}}{6} \quad \frac{-4 - \sqrt{232}}{6}$$

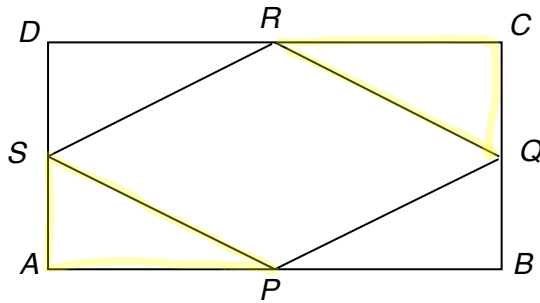
$$= 1.871924369$$

$$= 1.87$$

$$= -3.205257702$$

$$= -3.21$$

11. $ABCD$ is a rectangle. P , Q , R and S are the mid-points of the sides.



- (a) Prove that triangles APS and CRQ are congruent.

[3]

$$AP = RC \quad \text{and} \quad AS = CQ$$

$$\hat{SAP} = \hat{QCR}$$

no triangles are congruent (SAS)

- (b) Use your proof in part (a) to decide what is the special name given to the quadrilateral $PQRS$.

Give your reason.

[1]

$$SP = PQ = RQ = RS$$

Rhombus as the sides are equal

12. The square and the sector of a circle shown below have equal areas.

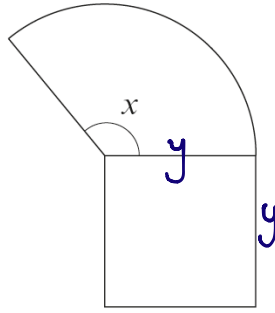


Diagram not drawn to scale

Calculate the size of angle x .

[3]

area of square = area of segment

$$y^2 = \frac{x}{360} \times \pi \times y^2$$

$$x = \frac{360}{\pi}$$

$$= 114.591559$$

$$= 115^\circ$$

13. (a) Express $\frac{x}{x-3} - \frac{x}{x+6}$ as a single fraction in its simplest form. [3]

$$\frac{x(x+6) - x(x-3)}{(x-3)(x+6)}$$

$$= \frac{x^2 + 6x - x^2 + 3x}{(x-3)(x+6)} = \frac{9x}{(x-3)(x+6)}$$

- (b) Simplify $\frac{49x^2 - 100}{14x + 20}$. [4]

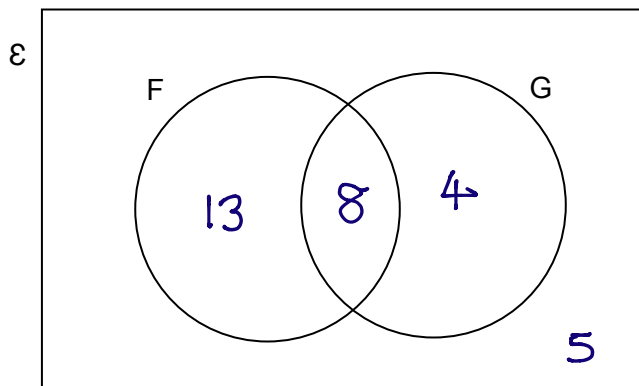
$$\frac{(7x-10)(\cancel{7x+10})}{2(\cancel{7x+10})} = \frac{7x-10}{2}$$

14. 30 students in a Year 11 class have decided which subjects they are going to study next year.

- 21 have decided to study French (F)
- 12 have decided to study German (G)
- 5 have decided not to study either French or German.

(a) Complete the Venn diagram below to show this information.
The universal set \mathcal{E} contains all the students in the class.

[2]



$$30 - 5 = 25$$

.....

(b) Given that a student, chosen at random, has decided to study French, what is the probability that this student has also decided to study German? [2]

.....
 $\frac{8}{21}$

$$\frac{\sqrt{3}}{3}$$

15. Circle the correct answer for each of the following questions.

(a) $\tan 30^\circ$ is equal to,

$$\frac{-1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{2}$$

$$\sqrt{3}$$

[1]

(b) $\cos 150^\circ$ is equal to,

$$-\frac{\sqrt{3}}{2}$$

$$\frac{1}{2}$$

$$\frac{\sqrt{3}}{2}$$

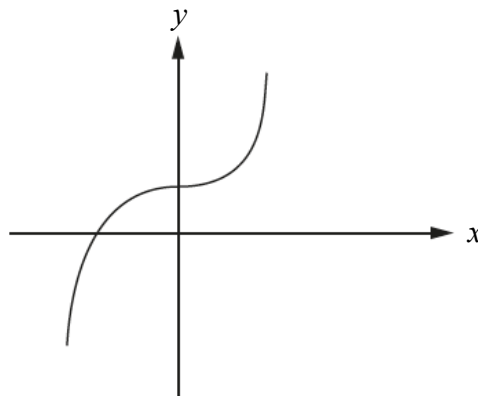
$$-\frac{1}{2}$$

$$-\frac{\sqrt{3}}{2}$$

$$\frac{1}{\sqrt{3}}$$

[1]

(c) The graph



can be represented by the equation,

$$y = ax^3 + b$$

$$y = ax^2 + b$$

$$y = ax + b$$

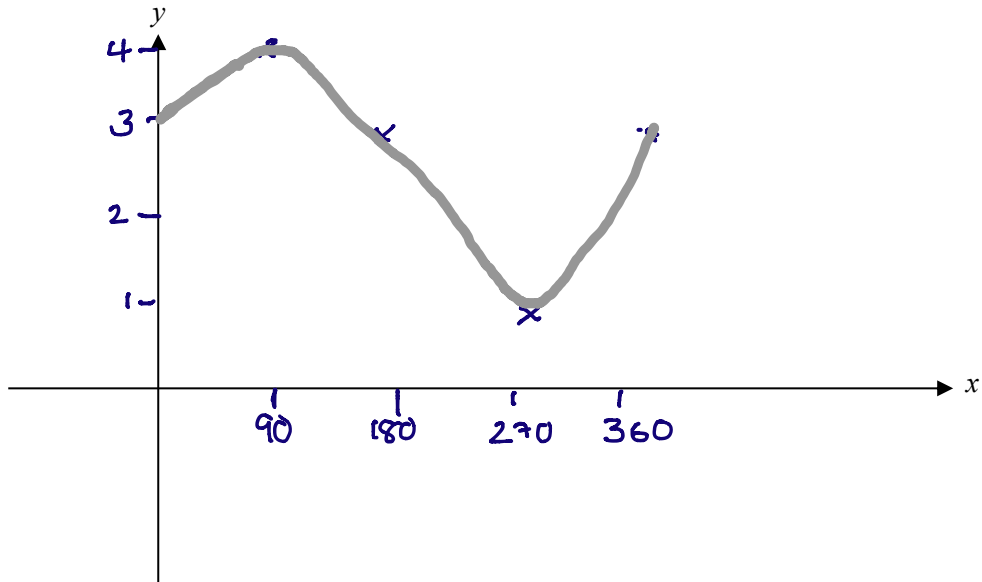
$$y = \frac{a}{x} + b$$

$$y = ax^2 + bx$$

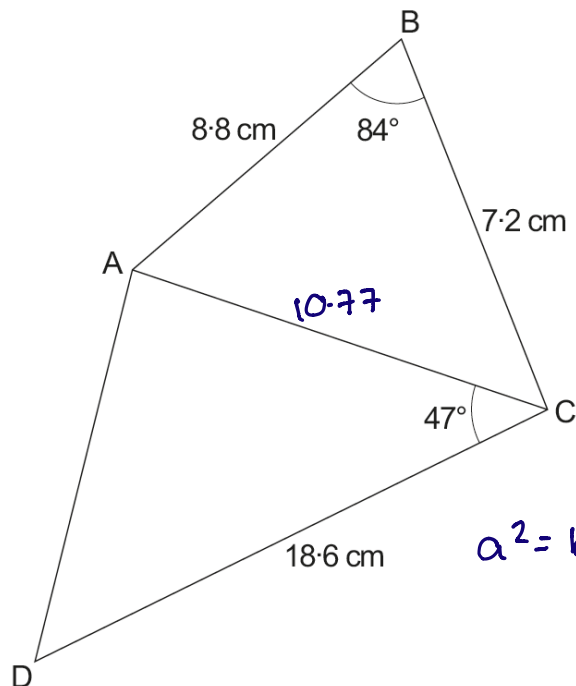
where a and b are both positive numbers.

[1]

16. Using the axes below, **sketch** the graph of $y = \sin x + 3$ for values of x from 0° to 360° . [2]



17.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

Diagram not drawn to scale

Calculate the area of triangle ACD.

[6]

$$\underline{AC} \quad AC^2 = 8.8^2 + 7.2^2 - 2 \times 8.8 \times 7.2 \times \cos 84$$

$$= 116.0341531$$

$$AC = \sqrt{116.0341531}$$

$$= 10.77191502 = 10.77 \text{ (2dp)}$$

$$\text{area} = \frac{1}{2} \times 10.77 \times 18.6 \sin 47$$

$$= 73.26614327 = 73.27 \text{ cm}^2 \text{ (2dp)}$$

18. A factory produces a very large number of beads which are either coloured red or coloured blue.
The beads are identical in all other respects.
The probability of a randomly chosen bead being red is 0.7.
The beads are randomly packed in boxes of 20 beads.

(a) What is the expected number of red beads in a box? [1]

$$0.7 \times 20 = 14$$

$$r = 14 \quad b = 6$$

(b) A particular box is known to contain the expected number of red and blue beads.
Two beads are chosen, at random, from this box without replacement.
Show that there is less than an 8% chance that both beads are blue. [3]

$$P(BB) = \frac{6}{20} \times \frac{5}{19} = \frac{30}{380} = 0.0789... = 7.89\%$$

which is less than 8%

(c) Two beads are chosen at random from the factory production line without replacement.
Will the probability that both beads are blue be the same as for part (b)?
You must justify your answer. [1]

$$P(B) = 0.3$$

$$P(B.B) = 0.3 \times 0.3 = 0.09$$

No.