Candidate Name	Centre Number					Candidate Number				
Mel@JustMaths						0				

SOLUTIONS

GCSE

MATHEMATICS - NUMERACY

UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

SPECIMEN PAPER SUMMER 2017

1 HOUR 45 MINUTES

ADDITIONAL MATERIALS
A calculator will be required for this paper.
A ruler, protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES
Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided in this booklet.

Take π as 3·14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

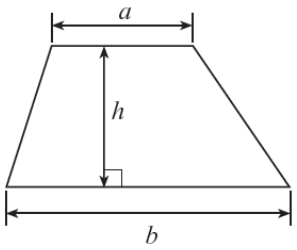
The number of marks is given in brackets at the end of each question or part-question.

The assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing in question **1**.

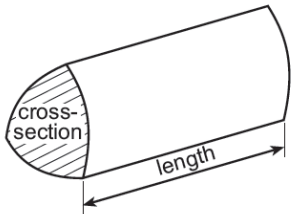
For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1.	**6**	
2.	**7**	
3.	**7**	
4.	**5**	
5.	**5**	
6.	**4**	
7.	**12**	
8.	**7**	
9.	**10**	
10.	**4**	
11	**13**	
TOTAL	**80**	

Formula list – Higher tier

Area of a trapezium = $\frac{1}{2}(a+b)h$

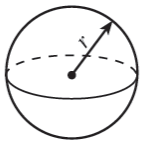


Volume of a prism = area of cross section × length



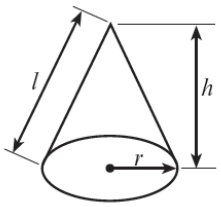
Volume of a sphere $= \frac{4}{3}\pi r^3$

Surface area of a sphere $= 4\pi r^2$



Volume of a cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of a cone $= \pi rl$

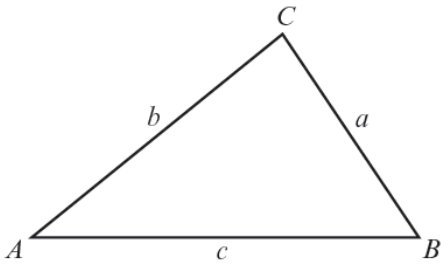


In any triangle ABC,

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.

1. *You will be assessed on the quality of your organisation, communication and accuracy in writing in this question*

Carys decides to invest £380 in a savings account for 6 years.
The account pays a rate of 2·54% AER.

Will Carys have sufficient money in her savings account to be able to buy a motor scooter costing £460 in 6 years' time?
You must show all your working and give a reason for your answer. [6]

100% + 2.54%

380×1.0254^6 = 102.54%

= 1.0254

= 441.7163504

= £441.72

No she won't have enough as its less than 460

2. Layla is investigating how much people would be prepared to pay for a bottle of water at an Eisteddfod.

Amount of money (£x)	Number of people	Mid	Fxm
$0 \leq x < 1$	12	0.5	6
$1 \leq x < 2$	44	1.5	66
$2 \leq x < 3$	20	2.5	50
$3 \leq x < 4$	4	3.5	14
	80		136

She asked a number of people at a concert on Monday how much they would be prepared to pay.

Monday's results are summarised in the table.

(a) Calculate an estimate for the mean amount of money that a person would be prepared to pay for a bottle of water. [4]

136 ÷ 80 = £1.70

(b) Monday was a cool day.
On Tuesday, it was much warmer.
Layla asked a further 60 people the same question as she did on Monday.
On Tuesday, the mean was £2.30.

Use the data collected over the two days to calculate an estimate for the mean amount of money that a person would be prepared to pay for a bottle of water.
Give your answer correct to the nearest penny. [3]

Mon Tues

80 60 (80 × 1.70) + (60 × 2.30)

= 274

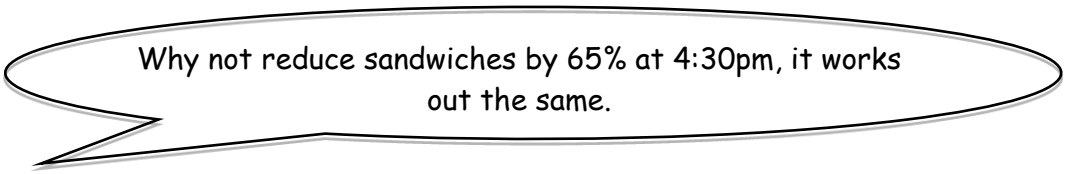
£1.70 £2.30 274 ÷ 140 = 1.9571428

= £1.96

3. Jane and Tomos own a sandwich business.

(a) They decide to price sandwiches individually each morning.
At 3 p.m. any unsold sandwiches are reduced by 45%.
Any sandwiches still unsold by 4:30p.m. are reduced by a further 20%.

Jane says



Tomos disagrees with Jane.

Using multipliers, show that Jane is incorrect. [4]

Jane: multiplier

= 100 - 65% = 35%

= 0·35

Tomos: 100% − 45% = 55%

! 0·55 × 0·8 = 0·44

(b) Write down and simplify two formulae, in terms of P, to calculate the reduced prices of sandwiches at 3 p.m. and at 4:30 p.m.
Let

- P be the full price of the sandwich.
- T be the price of a sandwich at 3p.m.
- R be the price of a sandwich after 4:30p.m. [3]

45% reduction

R 20% reduction

$T = P \times 0{\cdot}55$

$R = P \times 0{\cdot}44$

4.

Remember:

1 gallon = 8 pints

5 miles = 8 km

1 Litre = 1·75 pints

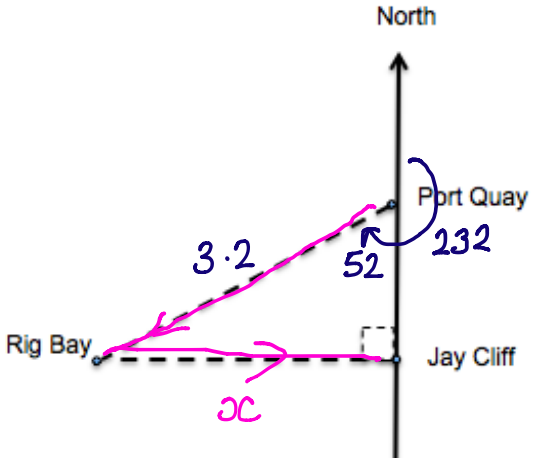
Lowri owns an old van.
It has an average fuel consumption of 7 km per litre.
Calculate an estimate for this fuel consumption in miles per gallon. [5]

7 km/litre $\Rightarrow 7 \times \frac{5}{8} = 4.375$ miles/litre

$4.375 \div 1.75 = 2.5$ miles/pint

$2.5 \times 8 = 20$ miles per gallon

5. The diagram shows the route a dolphin swam from Port Quay to Rig Bay and then to Jay Cliff.



232-180
= 52

Diagram not drawn to scale

Rig Bay is on a bearing of 232° from Port Quay.
The distance from Port Quay to Rig Bay is 3·2 km.
Calculate how far the dolphin swam altogether [5]

$\sin 52 = \frac{x}{3.2}$ $x = \sin 52 \times 3.2$

$= 2.521634412$

Total distance = 2·52 + 3·2 = 5·72 km

6. *NwyCymru* gas company uses the following formula to calculate how much to charge its customers:

charge (in pence) = (U × 11·546 + D × 31·48) × 1·05

The number of units of gas used by a customer is **U** and the number of days in the billing period is **D**.

A customer was charged £165.53 over a billing period of 90 days.
Calculate the number of gas units this customer used during this period. [4]

Charge = £165·53 = 16553p

U = ?

D = 90

$16553 = (U \times 11{\cdot}546 + 90 \times 31{\cdot}48) \times 1{\cdot}05$

$\frac{16553}{1{\cdot}05} = 11{\cdot}546U + 2833{\cdot}2$

$\frac{15764{\cdot}7619 - 2833{\cdot}2}{11{\cdot}546} = U$

U = 1120 units

7. *Pack4* is a company that makes cardboard boxes.

(a) One of their boxes, in the shape of a triangular prism, is shown below.

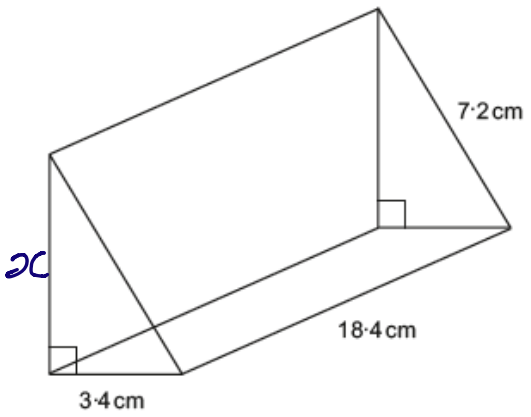


Diagram not drawn to scale

1 litre = $1000cm^3$
0·2L = $200cm^3$

A customer wants a box with a volume of 0·2 litres.

(i) State by how much the volume is greater or less than 0·2 litres, giving your answer in cm^3 correct to 2 significant figures. [6]

$x^2 = 7.2^2 - 3.4^2 = 40.28$ $x = \sqrt{40.28} = 6.346cm$

volume $= \left[\frac{1}{2} \times 3.4 \times 6.346\right] \times 18.4$

$= 198.523...$

Difference with 0·2 litre = $200 - 198.523 = 1.4767.. cm^3$

$= 1.5cm^3$

(ii) Explain why this may not be a suitable box for the customer. [1]

it may be the wrong shape

(b) Another of the cardboard boxes made by *Pack4* is a cuboid.
The cuboid measures 3·4 cm by 2·6 cm by 6·8 cm, where all measurements are correct to the nearest 1 mm.
By what percentage does the greatest possible volume of this cuboid exceed the least possible volume? [5]

mm cm

3·4cm = 34mm → UB 34·5 3·45

→ LB 33·5 3·35

2·6cm = 26mm → UB = 26·5 2·65

→ LB = 25·5 2·55

6·8cm = 68mm → UB = 68·5 6·85

→ LB = 67·5 6·75

UB of volume = $3{\cdot}45 \times 2{\cdot}65 \times 6{\cdot}85 = 62.626125$

LB of volume = $3{\cdot}35 \times 2{\cdot}55 \times 6{\cdot}75 = 57.661875$

Difference in volume = 4·96425

% = difference/least = $\frac{4{\cdot}96425}{57{\cdot}661875} \times 100 = 8{\cdot}6\%$

8. The following table gives areas and populations of 6 countries.

Country	Area (km^2)	Population in 2014	PopDensity
Wales	20 761	3 006 000	144·79
Singapore	716	5 399 200	7540·78
Bermuda	53	64 237	1212·02
India	3 287 240	1 244 392 079	378·55
Belgium	30 528	11 194 824	366·71
Tonga	720	104 270	144·82

(a) How many times as dense is the country with the greatest population density as the country with the least population density?
You must show all your working. [4]

Population density = $\frac{\text{population}}{\text{area}}$

Smallest = 144·79

largest = 7540·78

7540·78 ÷ 144·79

= 52·080...

= 52 times greater

(b) Which two countries have the same population densities to the nearest whole number of people per km^2? [1]
Circle your answer.

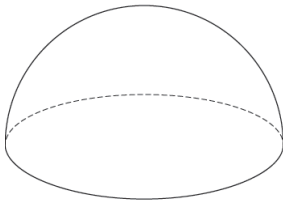
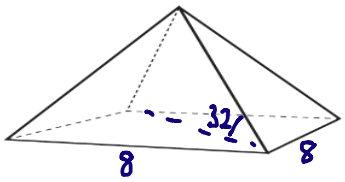
India and Belgium	Wales and Tonga	Singapore and Tonga	Wales and Belgium	Bermuda and Tonga

(c) If the information in the table had all been given correct to 2 significant figures would this make a difference to your answer in part *(a)*? [2]

Circle either TRUE or FALSE for each of the following statements.

No difference at all, the answer would be exactly the same.	TRUE	FALSE
One of the countries used in the comparison would be different.	TRUE	FALSE
Both countries used in the comparison would be different.	TRUE	FALSE
The only difference would be in rounding the final answer, nothing else in the calculation changes.	TRUE	FALSE
You cannot tell whether there would be a difference in the answer in part *(a)* if the information in the table had all been given correct to 2 significant figures.	TRUE	FALSE

9. *Blodyn Garden Products* makes caps for fence posts.

Solid metal fence caps	
	
Solid hemisphere cap	Solid pyramid cap

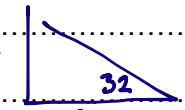
Blodyn Garden Products wants to make the price of the two different fence caps the same.
So it is important that the volume of metal used to make each cap is the same.

The lengths of the sides of the base of the pyramid are all 8 cm.
The angle between one of the sloping edges and the diagonal of the base is 32°.

(a) Calculate the height of the square-based pyramid cap. [5]

diagonal^2 = $8^2 + 8^2 = 64 + 64$

diagonal = $\sqrt{128}$ = 11.313708 5



$\tan 32 = \frac{x}{5.65685}$ $x = 5.66 \times \tan 32$

$= 3.534794..$

height = 3.5 cm (1dp)

÷2÷

(b) Calculate the volume of the square-based pyramid cap. [2]

volume of pyramid = $\frac{1}{3} \times 8 \times 8 \times 3.5...$

$= 75.409 cm^3$

(c) Calculate the radius of the hemispherical fence cap. [3]

$75 \cdot 409 = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{4}{6}\pi r^3$

$r^3 = \frac{75 \cdot 409 \times 6}{\pi \quad 4}$ $\quad r = \sqrt[3]{\frac{75 \cdot 406 \times 6}{4\pi}}$ $\quad r = 3 \cdot 3\text{cm}$

10. *(a)* A School Council wants to know pupils' views on their school uniform.
Which of the following statements shows how a truly random sample of the general population can be obtained? [1]
Circle your answer.

A: Randomly selecting pupils in the canteen at lunchtime.

B: Randomly selecting pupils from those that attend the next School Council meeting.

C: Randomly selecting pupils with a surname beginning with the letter J.

D: Giving each pupil a raffle ticket and then randomly drawing raffle tickets for selection.

E: Selecting every 2nd pupil from each form register.

(b) *VotePredict* is a specialist company working in the field of polling and predicting voting patterns in elections worldwide.
They are asked to organise a debate with an audience that is representative of five political parties.
The five political parties and their predicted number of votes, given in alphabetical order, are as follows.

Political Party	Predicted votes
Central	23 456
Economy	43 244
First Reformists	83 124
Status Quest	11 782
West Term	63 789

225395

The invited audience should be a stratified sample using this information.

It is intended to have 250 people in the audience at the debate.
How many people who intend to vote for the Central Party should be in the audience? [3]

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$\frac{23\,456}{225395} \times 250 \approx 26$ people

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11. Imran works for a company called *Derwen Insurance.*
His gross salary is £47 840 per year.

Below are extracts from HM Revenue and Customs and details of Imran's company pension scheme:

National Insurance contributions
• If you earn more than **£153** a week and up to **£805** a week, you pay 12% of the amount you earn between **£153** and **£805** • If you earn more than **£805** a week, you also pay 2% of all your earnings over **£805**

Source: HMRC 2014

Income tax threshold and rates	
Income tax threshold	£10,000 per year
Basic tax rate	20% on annual earnings above income tax threshold and up to £31,865
Higher tax rate	40% on annual earnings from £31,866 to £150,000
Additional tax rate	45% on annual earnings above £150,000

Source: HMRC 2014

***Derwen Insurance* Pension Scheme**			
Gross salary	**Contribution rate**	**Gross salary**	**Contribution rate**
Up to £13 500	5·5%	**£60 001 to £85 000**	9·9%
£13 501 to £21 000	5·8%	**£85 001 to £100 000**	10·5%
£21 001 to £34 000	6·5%	**£100 001 to £150 000**	11·4%
£34 001 to £43 000	6·8%	**£150 001 or more**	12·5%
£43 001 to £60 000	8·5%		

Using the information on the previous page, calculate Imran's **weekly** net salary.
You must show all your working. [13]

Salary 47840 per year ÷52 = 920 per week

NI earns more than 153pw 12% of 805-153 = 0·12 × 652 = 78·24

2% of 920-805 = 0·02 × 115 = 2·30

Tax 0-10,000 = no tax

20% on 31865-10,000 = 0·2 × 21865 = 4373

40% on 47840-31865 = 0·4 × 15975 = 6390

10763 per year = 206·98 per week.

Pension 8·5% of 47840 = 0·085 × 47840 = 4066·40 per year = 78·2 per week

Net Salary = 920 - (78·24 + 2·30 + 206·98 + 78·2)

= £554·28