Algebraic Fractions (H)

A collection of 9-1 Maths GCSE Sample and Specimen questions from AQA, OCR, Pearson-Edexcel and WJEC Eduqas.

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Total Marks:	

1. a) Express as a single fraction.

$$\frac{m+1}{n+1} - \frac{m}{n}$$

   Simplify your answer.

$$\frac{n(m+1) - m(n+1)}{n(n+1)} = \frac{mn + n - mn - m}{n(n+1)}$$

$$\frac{n-m}{n(n+1)}$$

a) ......$\frac{n-m}{n(n+1)}$...... [2]

b) Using your answer to part (a), prove that if m and n are positive integers and m < n, then

$$\frac{m+1}{n+1} - \frac{m}{n} > 0$$

if $m < n$ $n - m > 0$

$n(n+1) > 0$ $\therefore$ $\frac{n-m}{n(n+1)} > 0$

[2]

2. Show that $\dfrac{1}{6x^2 + 7x - 5} \div \dfrac{1}{4x^2 - 1}$ simplifies to $\dfrac{ax+b}{cx+d}$ where a, b, c and d are integers.

$6x^2 + 7x - 5 = (3x+5)(2x-1)$

$4x^2 - 1 = (2x+1)(2x-1)$

$\dfrac{1}{(3x+5)(2x-1)} \times \dfrac{(2x+1)(2x-1)}{1} = \dfrac{2x+1}{3x+5}$

$a = 2 \quad b = 1$
$c = 3 \quad d = 5$

[3]

3. Write

$$4 - \left[(x+3) \div \frac{x^2 + 5x + 6}{x - 2}\right]$$

$(x+2)(x+3)$

$$4 - \left[\frac{x+3}{1} \times \frac{x-2}{(x+2)(x+3)}\right]$$

as a single fraction in its simplest form.
You must show your working.

$$4 - \left[\frac{(x+3)(x-2)}{(x+2)(x+3)}\right] = 4 - \frac{x-2}{x+2} = \frac{4(x+2) - (x-2)}{x+2}$$

$$= \frac{4x + 8 - x + 2}{x+2} = \frac{3x + 10}{x+2}$$

[4]

4. Given that

$$2x - 1 : x - 4 = 16x + 1 : 2x - 1$$

find the possible values of x.

$$\frac{2x-1}{x-4} = \frac{16x+1}{2x-1}$$

$$(2x-1)(2x-1) = (16x+1)(x-4)$$

$$4x^2 - 4x + 1 = 16x^2 - 63x - 4$$

$$12x^2 - 59x - 5 = 0$$

$$(12x + 1)(x - 5) = 0$$

$$x = -\frac{1}{12} \qquad x = 5$$

[5]

5. Show that $\frac{a}{b+1} - \frac{a}{(b+1)^2}$ can be written as $\frac{ab}{(b+1)^2}$

$$\frac{a(b+1)}{(b+1)(b+1)} - \frac{a}{(b+1)(b+1)} = \frac{a(b+1) - a}{(b+1)^2} = \frac{ab + a - a}{(b+1)^2}$$

$$= \frac{ab}{(b+1)^2} \qquad QED$$

[2]

6. Show that $\frac{3x+6}{x^2 - 3x - 10} \div \frac{x+5}{x^3 - 25x}$ simplifies to ax where a is an integer.

$$3x + 6 = 3(x+2)$$

$$x^2 - 3x - 10 = (x+2)(x-5)$$

$$x^3 - 25x = x(x^2 - 25) = x(x+5)(x-5)$$

$$\frac{3(x+2)}{(x+2)(x-5)} \times \frac{x(x+5)(x-5)}{x+5} = \frac{3x(x-5)}{(x-5)} = 3x$$

QED

[4]

7. Solve $\frac{x+2}{3x} + \frac{x-2}{2x} = 3$

$\frac{2x(x+2) + 3x(x-2)}{6x^2} = 3$

$2x^2 + 4x + 3x^2 - 6x = 18x^2$

$5x^2 - 2x = 18x^2$

$13x^2 + 2x = 0$

$x(13x + 2) = 0$

$x = 0$ $x = \frac{-2}{13}$

$x =$ .... [3]

8. Show that $\frac{2x^2 - 3x - 5}{x^2 + 6x + 5}$ can be written in the form $\frac{ax + b}{cx + d}$ where a, b, c and d are integers.

$2x^2 - 3x - 5 = (2x - 5)(x + 1)$

$x^2 + 6x + 5 = (x + 5)(x + 1)$

$\Rightarrow \frac{(2x - 5)(x + 1)}{(x + 5)(x + 1)} = \frac{2x - 5}{x + 5}$

$\therefore a = 2 \quad b = -2$
$c = 1 \quad d = 5$

[3]

9. Show that $\frac{2w + 4}{w^2 - 25} \times \frac{w + 5}{w^2 + 3w + 2} \times (3w^2 - 16w + 5)$

$\hookrightarrow (3w - 1)(w - 5)$

Simplifies to $\frac{aw + b}{cw + d}$ where a, b, c and d are integers.

$2w + 4 = 2(w + 2) \quad w^2 - 25 = (w + 5)(w - 5)$

$w^2 + 3w + 2 = (w + 1)(w + 2)$

$\frac{2(w + 2)}{(w + 5)(w - 5)} \times \frac{(w + 5)}{(w + 1)(w + 2)} \times \frac{(3w - 1)(w - 5)}{1}$

$= \frac{2(3w - 1)}{w + 1} = \frac{6w - 2}{w + 1}$

$a = 6 \quad b = -2$
$c = 1 \quad d = 1$

[5]

10. Show that

$$\frac{4}{x - 3} - \frac{2}{x + 1} = \frac{2(x + 5)}{(x - 3)(x + 1)}$$

$\frac{4(x + 1) - 2(x - 3)}{(x - 3)(x + 1)} = \frac{4x + 4 - 2x + 6}{(x - 3)(x + 1)}$

$= \frac{2x + 10}{(x - 3)(x + 1)} = \frac{2(x + 5)}{(x - 3)(x + 1)}$

[3]

11. Show that $\dfrac{2x+1}{3} + \dfrac{5x-2}{2}$ simplifies to $\dfrac{19x-4}{6}$

$$\frac{2(2x+1) + 3(5x-2)}{6}$$

[2]

$$= \frac{4x + 2 + 15x - 6}{6}$$

$$= \frac{19x - 4}{6}$$

CREDITS AND NOTES

Question	Awarding Body	Question	Awarding Body
1	OCR	8	Pearson Edexcel
2	Pearson Edexcel	9	AQA
3	Pearson Edexcel	10	OCR
4	Pearson Edexcel	11	AQA
5	Pearson Edexcel		
6	Pearson Edexcel		
7	Pearson Edexcel		

Notes:

These questions have been retyped from the original sample/specimen assessment materials and whilst every effort has been made to ensure there are no errors, any that do appear are mine and not the exam board s (similarly any errors I have corrected from the originals are also my corrections and not theirs!).

Please also note that the layout in terms of fonts, answer lines and space given to each question does not reflect the actual papers to save space.

These questions have been collated by me as the basis for a GCSE working party set up by the GLOW maths hub - if you want to get involved please get in touch. The objective is to provide support to fellow teachers and to give you a flavour of how different topics "could" be examined. They should not be used to form a decision as to which board to use. There is no guarantee that a topic will or won't appear in the "live" papers from a specific exam board or that examination of a topic will be as shown in these questions.

Links:

AQA http://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300

OCR http://ocr.org.uk/gcsemaths

Pearson Edexcel http://qualifications.pearson.com/en/qualifications/edexcel-gcses/mathematics-2015.html

WJEC Eduqas http://www.eduqas.co.uk/qualifications/mathematics/gcse/

Contents:

This version contains questions from:

AQA – Sample Assessment Material, Practice set 1 and Practice set 2

OCR – Sample Assessment Material and Practice set 1

Pearson Edexcel – Sample Assessment Material, Specimen set 1 and Specimen set 2

WJEC Eduqas – Sample Assessment Material