

Algebraic Proof (H)

A collection of 9-1 Maths GCSE Sample and Specimen questions from AQA, OCR, Pearson-Edexcel and WJEC Eduqas.

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Total Marks:	

1. Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n .

$$\begin{aligned}(2n+1)^2 &= (2n+1)(2n+1) \\ &= 4n^2 + 2n + 2n + 1\end{aligned}$$

$$-(2n+1) = -2n - 1$$

$$\begin{aligned}4n^2 + 4n + 1 - 2n - 1 &= 4n^2 + 2n \\ &= 2n(2n+1)\end{aligned}$$

$2n$ is even $2n+1$ is odd
even \times odd = even.
 $\therefore (2n+1)^2 - (2n+1)$ is even.

[3]

2. c is a positive integer.

Prove that $\frac{6c^3 + 30c}{3c^2 + 15}$ is an even number.

$$\frac{6c(c^2 + 5)}{3(c^2 + 5)} = \frac{6c}{3} = 2c$$

$2c$ will always be even, therefore $\frac{6c^3 + 30c}{3c^2 + 15}$ is always even [3]

3. a) Prove that the sum of four consecutive whole numbers is always even.

let first number be n

$$\begin{aligned}n + (n+1) + (n+2) + (n+3) &= 4n + 6 \\ &= 2(2n+3)\end{aligned}$$

this is a multiple of 2, therefore will be even [3]

b) Give an example to show that the sum of four consecutive integers is not always divisible by 4.

1st attempt $4 + 5 + 6 + 7 = 20$ even and divisible by 4

2nd attempt $12 + 13 + 14 + 15 = 54$ is even but not divisible by 4

[2]

The n th term of the sequence is $6n+1$

4. Here are the first five terms of an arithmetic sequence.

let \rightarrow 1st term = $6n+1$ and $6m+1$ be any other

Prove that the difference between the squares of any two terms of the sequence is always a multiple of 24 $p^2 - q^2$ & difference of any 2 squares

$$(6m+1)^2 = 36m^2 + 12m + 1$$

$$(6n+1)^2 = 36n^2 + 12n + 1$$

$$\text{Difference} = (36n^2 + 12n + 1) - (36m^2 + 12m + 1)$$

$$= 36n^2 - 36m^2 + 12n - 12m$$

$$= 36(n^2 - m^2) + 12(n - m)$$

$$= 12[3(n^2 - m^2) + n - m]$$

Given the terms of the sequence are all odd

$$n^2 = \text{even} \quad m^2 = \text{even} \quad \text{odd} - \text{odd}$$

$$3(\text{even} - \text{even}) + \text{even}$$

$$3(\text{even} + \text{even}) = 3(\text{even}) \text{ which will always be even} \\ + \text{so a multiple of 2.} \quad [6]$$

$$12 \times (\text{a multiple of 2})$$

5. The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

let n be the first number

$$\text{product} \Rightarrow n \times (n+1) = n^2 + n$$

$$\text{so:- } n^2 + n + n + 1 = n^2 + 2n + 1$$

$$\begin{aligned} & (n+1)(n+1) \\ &= (n+1)^2 \end{aligned}$$

which is the square of the larger number

[3]

6. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

let n be the first number.

$n+1$ is the second number.

$$(n+1)^2 - n^2$$

$$= n^2 + 2n + 1 - n^2$$

$$= 2n + 1$$

$$\text{OR } n + n + 1$$

which is the sum of the two integers.

[4]

CREDITS AND NOTES

Question	Awarding Body
1	Pearson Edexcel
2	AQA
3	OCR
4	Pearson Edexcel
5	Pearson Edexcel
6	Pearson Edexcel

Notes:

These questions have been retyped from the original sample/specimen assessment materials and whilst every effort has been made to ensure there are no errors, any that do appear are mine and not the exam board's (similarly any errors I have corrected from the originals are also my corrections and not theirs!).

Please also note that the layout in terms of fonts, answer lines and space given to each question does not reflect the actual papers to save space.

These questions have been collated by me as the basis for a GCSE working party set up by the GLOW maths hub - if you want to get involved please get in touch. The objective is to provide support to fellow teachers and to give you a flavour of how different topics "could" be examined. They should not be used to form a decision as to which board to use. There is no guarantee that a topic will or won't appear in the "live" papers from a specific exam board or that examination of a topic will be as shown in these questions.



Links:

AQA <http://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300>

OCR <http://ocr.org.uk/gcsemaths>

Pearson Edexcel <http://qualifications.pearson.com/en/qualifications/edexcel-gcses/mathematics-2015.html>

WJEC Eduqas <http://www.eduqas.co.uk/qualifications/mathematics/gcse/>

Contents:

This version contains questions from:

AQA – Sample Assessment Material, Practice set 1 and Practice set 2

OCR – Sample Assessment Material and Practice set 1

Pearson Edexcel – Sample Assessment Material, Specimen set 1 and Specimen set 2

WJEC Eduqas – Sample Assessment Material