

Iteration (H)

A collection of 9-1 Maths GCSE Sample and Specimen questions from AQA, OCR, Pearson-Edexcel and WJEC Eduqas.

the process mark will be awarded for substituting $x=0$ and $x=1$ into the equation BUT, to get full marks you need to prove there is a change of sign

Name:	Mel@JustMaths
Total Marks:	

1 (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

$$x^3 + 4x - 1 = 0$$

when $x = 0$ $0^3 + 4(0) - 1 = -1$
 when $x = 1$ $1^3 + 4(1) - 1 = 4$ } There is a change in sign \therefore there is a solution between 0 and 1 [2]

b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

$$-x^3 \quad x^3 + 4x = 1 - x^3$$

$$4x = 1 - x^3$$

$$\frac{4x}{4} = \frac{1 - x^3}{4}$$

$$x = \frac{1}{4} - \frac{x^3}{4} \quad \text{QED} \quad [1]$$

c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

$$x_0 = 0$$

$$x_1 = \frac{1}{4} - \frac{0^3}{4}$$

$$= \frac{1}{4}$$

$$= 0.25$$

$$x_2 = \frac{1}{4} - \frac{0.25^3}{4}$$

$$= \underline{0.246}$$

[3]

2. An approximate solution to an equation is found using this iterative process.

$$x_{n+1} = \frac{(x_n)^3 - 3}{8} \text{ and } x_1 = -1$$

a) Work out the values of x_2 and x_3

$$x_2 = \frac{(-1)^3 - 3}{8} = -\frac{4}{8} = -0.5$$

$$x_3 = \frac{(-0.5)^3 - 3}{8} = -0.390625$$

$$x_2 = \underline{-0.5}$$

$$x_3 = \underline{-0.390625} \quad [2]$$

b) Work out the solution to 6 decimal places.

$$x_4 = -0.38245058$$

$$x_7 = -0.38196609$$

$$x_5 = -0.381992556$$

$$x_8 = -0.381966015$$

$$x_6 = -0.381967463$$

$$x = -0.381966 \quad [1]$$

3. a) Show that the equation $3x^2 - x^3 + 3 = 0$ can be rearranged to give

$$x = 3 + \frac{3}{x^2}$$

$$3x^2 - x^3 + 3 = 0$$

$$(\div x^2) \quad 3x^2 + 3 = x^3$$

$$3 + \frac{3}{x^2} = x$$

$$x = 3 + \frac{3}{x^2} \quad \text{Q.E.D.}$$

[2]

b) Using

$$x_{n+1} = 3 + \frac{3}{x_n^2} \text{ with } x_0 = 3.2$$

find the values of x_1 , x_2 and x_3

$$x_1 = 3 + \frac{3}{3.2^2} = 3.29296875$$

$$x_2 = 3 + \frac{3}{3.29296875^2} = 3.276659786$$

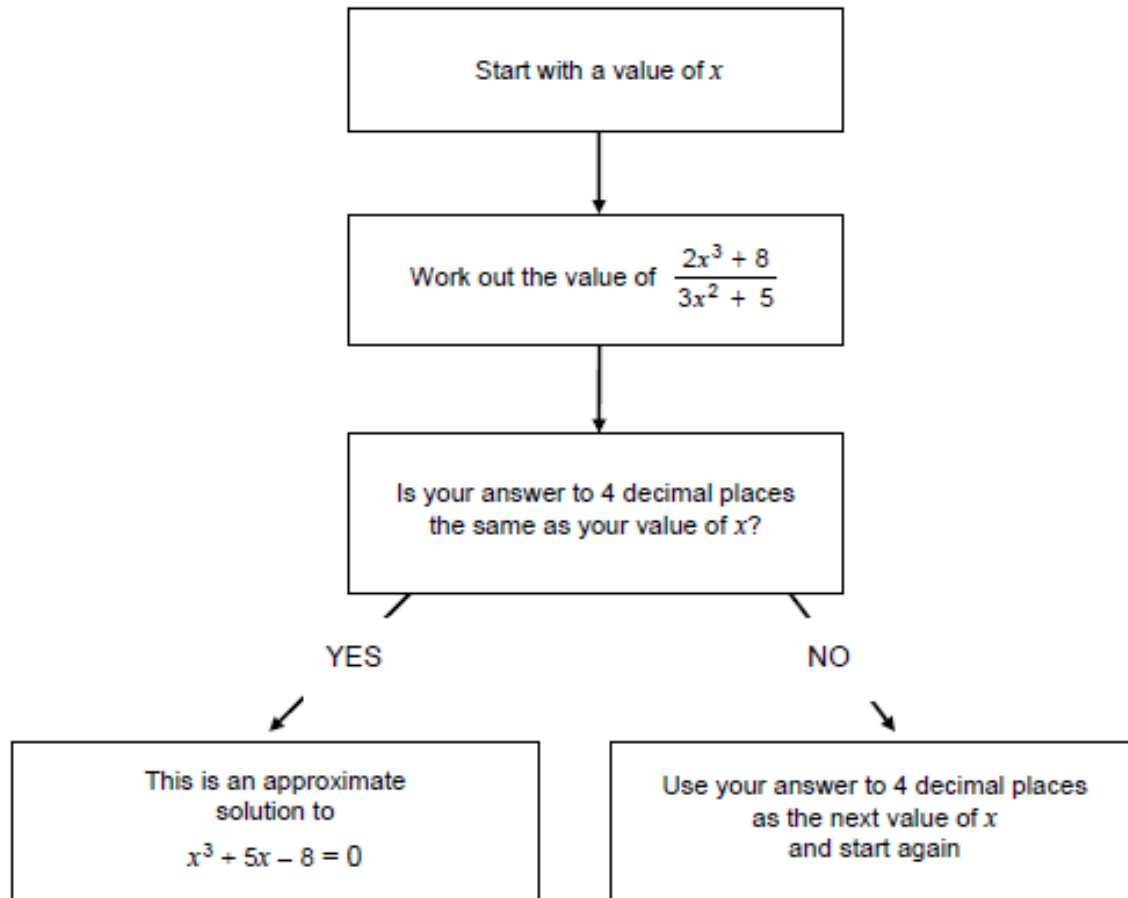
$$x_3 = 3 + \frac{3}{3.276659786^2} = 3.279420685$$

..... [3]

c) Explain what the values of x_1 , x_2 and x_3 represent.

they represent an increasingly accurate estimate
to $3x^2 - x^3 + 3 = 0$ [1]

4. This iterative process can be used to find approximate solutions to $x^3 + 5x - 8 = 0$



a) Use this iterative process to find a solution to 4 decimal places of $x^3 + 5x - 8 = 0$

Start with the value $x = 1$

$$x = 1 \quad \frac{2 \times 1^3 + 8}{3 \times 1^2 + 5} = 1.25$$

$$x = 1.25 \quad = 1.229032258$$

$$x = 1.229 \quad = 1.228860251$$

$$x = 1.2289 \quad = 1.228860244$$

$$\therefore x = 1.2289$$

[3]

b) By substituting your answer to part (a) into $x^3 + 5x - 8$

comment on the accuracy of your solution to $x^3 + 5x - 8 = 0$

$$1.2289^3 + 5 \times 1.2289 - 8 = 0.000378893$$

This solution is relatively close to zero, so can be considered a close estimate

[2]

5. a) Complete the table for $y = x^3 - 6x - 5$.

x	0	1	2	3	4
y	-5	-10	-9	4	35

$$0^3 - 6(0) - 5$$

$$4^3 - 6(4) - 5$$

[2]

b) (i) Between which two consecutive integers is there a solution to the equation

$$x^3 - 6x - 5 = 0?$$

Give a reason for your answer.

A solution lies between $x = 2$ and $x = 3$

Because the y value has a change in sign from negative to positive and therefore crosses the x -axis (it has a solution) [2]

(ii) Choose a value of x between the two values you gave in part (b)(i).

Calculate the corresponding value of y . (midpoint of 2 and 3)

$$2.5^3 - 6(2.5) - 5$$

$$(b)(ii) x = 2.5$$

$$y = -4.375$$
 [2]

(iii) State a smaller interval in which the solution lies.

$$(iii) 2.5 < x < 3$$
 [1]

6. A sequence of numbers is formed by the iterative process $a_{n+1} = (a_n)^2 - a_n$

a) Describe the sequence of numbers when $a_1 = 1$

Show working to justify your answer.

$$a_2 = 1 - 1 = 0 \quad \therefore \text{when } a_1 = 1 \text{ the sequence ends with repeated zeros [1]}$$

$$a_3 = 0 - 0 = 0$$

b) Describe the sequence of numbers when $a_1 = -1$

Show working to justify your answer.

$$a_2 = (-1)^2 - 1 = 2 \quad \therefore \text{when } a_1 = -1 \text{ the sequence ends with repeated '2's [2]}$$

$$a_3 = (2)^2 - 2 = 2$$

c) Work out the value of a_2 when $a_1 = 1 - \sqrt{2}$

$$a_2 = (1 - \sqrt{2})^2 - (1 - \sqrt{2})$$

$$= (1 - \sqrt{2})(1 - \sqrt{2}) - 1 + \sqrt{2}$$

$$= 1 - \sqrt{2} - \sqrt{2} + 2 - 1 + \sqrt{2}$$

$$= \underline{\underline{2 - \sqrt{2}}}$$

CREDITS AND NOTES

Question	Awarding Body
1	Pearson Edexcel
2	AQA
3	Pearson Edexcel
4	AQA
5	OCR
6	AQA

Notes:

These questions have been retyped from the original sample/specimen assessment materials and whilst every effort has been made to ensure there are no errors, any that do appear are mine and not the exam board's (similarly any errors I have corrected from the originals are also my corrections and not theirs!).

Please also note that the layout in terms of fonts, answer lines and space given to each question does not reflect the actual papers to save space.

These questions have been collated by me as the basis for a GCSE working party set up by the GLOW maths hub - if you want to get involved please get in touch. The objective is to provide support to fellow teachers and to give you a flavour of how different topics "could" be examined. They should not be used to form a decision as to which board to use. There is no guarantee that a topic will or won't appear in the "live" papers from a specific exam board or that examination of a topic will be as shown in these questions.



Links:

AQA <http://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300>

OCR <http://ocr.org.uk/gcsemaths>

Pearson Edexcel <http://qualifications.pearson.com/en/qualifications/edexcel-gcses/mathematics-2015.html>

WJEC Eduqas <http://www.eduqas.co.uk/qualifications/mathematics/gcse/>

Contents:

This version contains questions from:

AQA – Sample Assessment Material, Practice set 1 and Practice set 2

OCR – Sample Assessment Material and Practice set 1

Pearson Edexcel – Sample Assessment Material, Specimen set 1 and Specimen set 2

WJEC Eduqas – Sample Assessment Material