Iteration (H)

A collection of 9-1 Maths GCSE Sample and Specimen questions from AQA, OCR, Pearson-Edexcel and WJEC Eduqas.

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Total Marks:	

1 (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

$x = 0 \Rightarrow 0^3 + 0 = 0$ Too Low

$x = 1 \Rightarrow 1^3 + 4 = 5$ Too high

$\therefore$ Solution between 0 & 1

[2]

b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

$-x^3 \quad -x^3$

$4x = 1 - x^3$

$\div 4 \quad \div 4$

$x = \frac{1}{4} - \frac{x^3}{4}$

[1]

c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{{x_n}^3}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

$x_0 = 0$

$x_1 = \frac{1}{4} - \frac{0^3}{4}$

$= 0.25$

$x_2 = \frac{1}{4} - \frac{0.25^3}{4}$

$= 0.246$

[3]

2. An approximate solution to an equation is found using this iterative process.

$$x_{n+1} = \frac{(x_n)^3 - 3}{8} \text{ and } x_1 = -1$$

a) Work out the values of x_2 and x_3

$x_2 = \frac{(-1)^3 - 3}{8}$

$x_3 = \frac{(-0.5)^3 - 3}{8}$

$x_2 =$ -0.5

$x_3 =$ -0.390625 [2]

b) Work out the solution to 6 decimal places.

$x_4 = -0.38245058$

$x_5 = -0.381992556$

$x_6 = -0.381967463$

$x_7 = -0.38196609$

$x_8 = -0.381966015$

$x = -0.381966$

[1]

3. a) Show that the equation $3x^2 - x^3 + 3 = 0$ can be rearranged to give

$$x = 3 + \frac{3}{x^2}$$

$+x^3$ $+x^3$

$x^3 = 3x^2 + 3$

$\div x^2$ $\div x^2$

$x = 3 + \frac{3}{x^2}$

[2]

b) Using

$x_{n+1} = 3 + \frac{3}{x_n^2}$ with $x_0 = 3.2$

find the values of x_1, x_2 and x_3

$x_1 = 3 + 3/3.2^2 = 3.29296875$

$x_2 = 3.276659786$

$x_3 = 3.279420685$

.... [3]

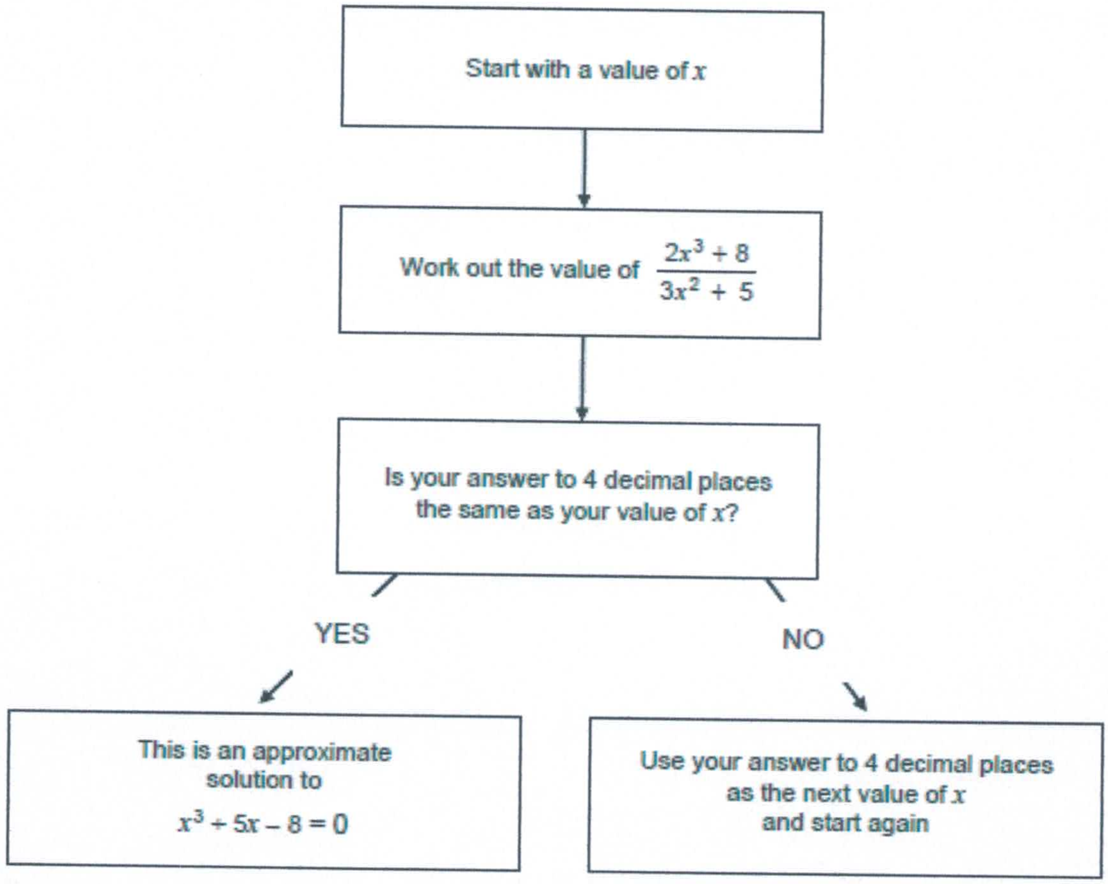
c) Explain what the values of x_1, x_2 and x_3 represent.

Increasingly accurate estimations of one solution to

$3x^2 - x^3 + 3 = 0$

[1]

4. This iterative process can be used to find approximate solutions to $x^3 + 5x - 8 = 0$



a) Use this iterative process to find a solution to 4 decimal places of $x^3 + 5x - 8 = 0$

Start with the value $x = 1$

$x = 1 \quad \frac{2 \times 1^3 + 8}{3 \times 1^2 + 5} = 1.25$

$x = 1.25 \quad \Rightarrow 1.229032258$

$x = 1.229 \quad \Rightarrow 1.228860251$

$x = 1.2289 \quad \Rightarrow 1.228860244$

$x = 1.2289$

[3]

b) By substituting your answer to part (a) into $x^3 + 5x - 8$

comment on the accuracy of your solution to $x^3 + 5x - 8 = 0$

$1.2289^3 + 5 \times 1.2289 - 8 = 0.000378893$

Close to zero, so good estimate.

[2]

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5. a) Complete the table for $y = x^3 - 6x - 5$.

x	0	1	2	3	4
y	-5	-10	-9	4	35

$0^3 - 6 \times 0 - 5$
$= -5$

$4^3 - 6 \times 4 - 5$
$= 35$

[2]

b) (i) Between which two consecutive integers is there a solution to the equation

$x^3 - 6x - 5 = 0$?

Give a reason for your answer.

A solution lies between x = ...2...... and x = ...3......

Because ...the y value changes from negative to positive therefore crossing the x-axis ($y = 0$)... [2]

(ii) Choose a value of x between the two values you gave in part (b)(i).

Calculate the corresponding value of y .

$2.5^3 - 6 \times 2.5 - 5$

(b)(ii) x = ...2.5......

y = ...-4.375...... [2]

(iii) State a smaller interval in which the solution lies.

(iii) ......$2.5 < x < 3$...... [1]

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6. A sequence of numbers is formed by the iterative process $a_{n+1} = (a_n)^2 - a_n$

a) Describe the sequence of numbers when $a_1 = 1$

Show working to justify your answer.

$a_2 = 1 - 1 = 0$ When $a_1 = 1$ the sequence

$a_3 = 0 - 0 = 0$ ends in repeated 0's

[1]

b) Describe the sequence of numbers when $a_1 = -1$

Show working to justify your answer.

$a_2 = (-1)^2 - -1 = 2$ When $a_1 = -1$ the sequence

$a_3 = 2^2 - 2 = 2$ ends in repeated 2

[2]

c) Work out the value of a_2 when $a_1 = 1 - \sqrt{2}$

$a_2 = (1-\sqrt{2})^2 - (1-\sqrt{2})$

$a_2 = (1-\sqrt{2})(1-\sqrt{2}) - 1 + \sqrt{2}$

[2]

	1	$-\sqrt{2}$
1	1	$-\sqrt{2}$
$-\sqrt{2}$	$-\sqrt{2}$	2

$= 1 - \sqrt{2} - \sqrt{2} + 2 - 1 + \sqrt{2}$

$= \underline{2 - \sqrt{2}}$

or

$a_2 = (1-\sqrt{2})(1-\sqrt{2}) - 1(1-\sqrt{2}$

$= (1-\sqrt{2})(1-\sqrt{2}-1)$

$= (1-\sqrt{2}) \times -\sqrt{2}$

$= \underline{2-\sqrt{2}}$