Similarity & Congruence (H)

A collection of 9-1 Maths GCSE Sample and Specimen questions from AQA, OCR, Pearson-Edexcel and WJEC Eduqas.

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Total Marks:	

1. Triangle ABC is isosceles with $AB = AC$.

   The line BP bisects $A\hat{B}C$.

   The line CQ bisects $A\hat{C}B$.

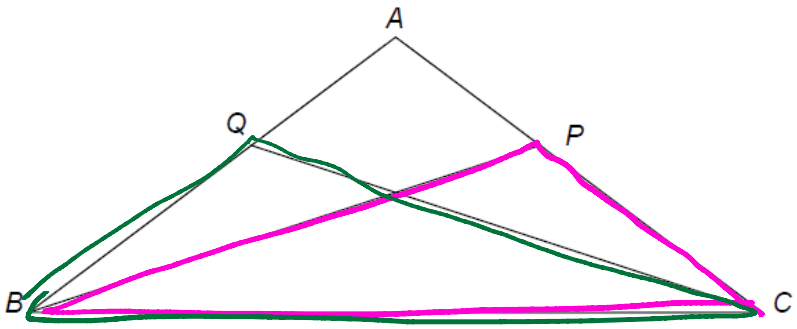


Diagram not drawn to scale

   Prove that triangle BCP and triangle CBQ are congruent.

   You must give reasons to support your statements.

PCB = QBC 2 angles in an isosceles triangle (ABC) are equal
BC = shared side so is equal length in each triangle
PBC = QCB angles were bisected ∴ triangles are congruent (ASA) [5]

2. Steph is solving a problem.

   Cube A has a surface area of 150 cm^2
   Cube B has sides half the length of cube A
   What is the volume of cube B?

SA = 150 cm2

1 face = 150 ÷ 6 = 25m^2

side length = $\sqrt{25}$ = 5cm

B side length = 2.5 cm

Volume = 2.5^3

   To solve this problem, Steph decides to

   3 halve the ~~surface area~~ side length
   2 calculate the square root of the answer
   1 then divide by 6
   4 then cube this answer to work out the volume.

   Evaluate Steph's method.

Steps method is wrong (1) you halve the side length not the area (2) the order is incorrect. [2]

3. Which of these is not used to prove that triangles are congruent?

Circle your answer.

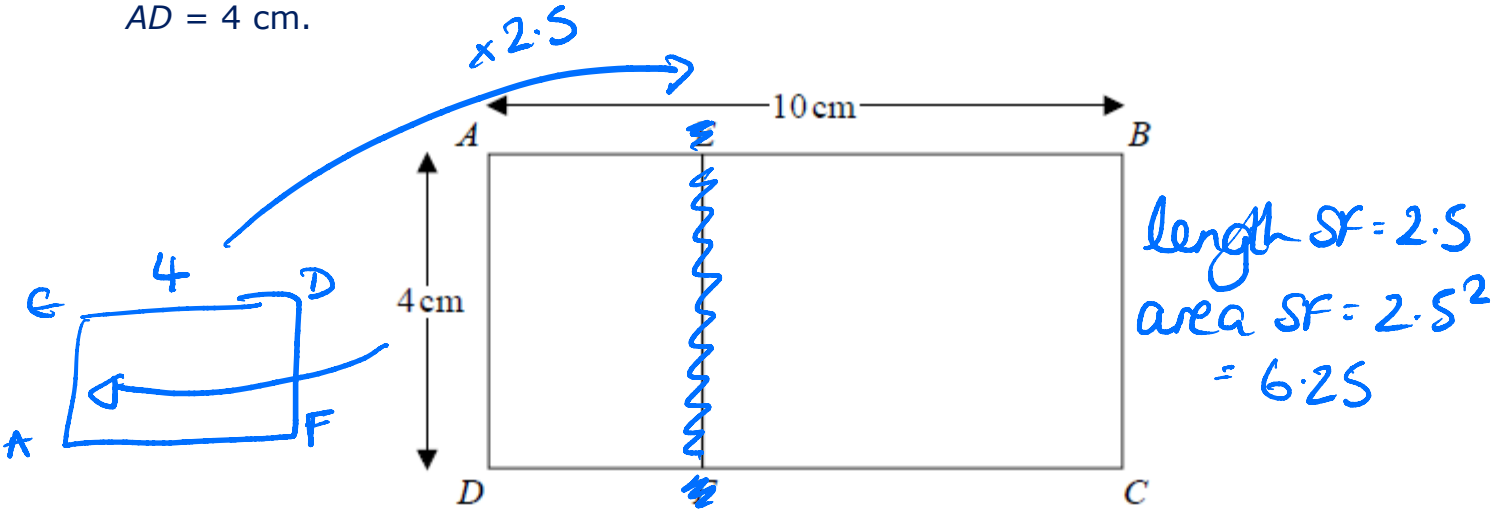
SSS SAS AAA RHS

[1]

4. Rectangle *ABCD* is mathematically similar to rectangle DAEF.

AB = 10 cm.

AD = 4 cm.



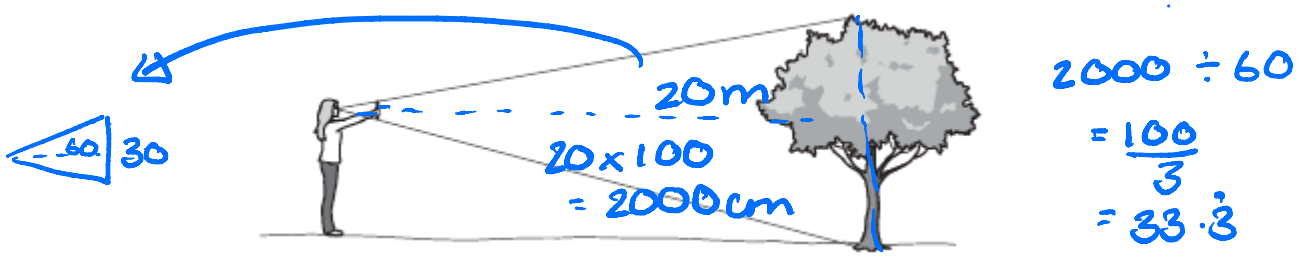
Work out the area of rectangle *DAEF*.

area of ABCD = 4 × 10 = 40cm²

area of AEDF = 40 ÷ 2·5² = 40 ÷ 6·25

6·4 ..... cm² [3]

5. (a) Anna estimates the height of a tree.



Anna holds a ruler vertically so the height of the tree is exactly covered by the ruler.

She is 20 metres from the tree.
The ruler is 30 cm long.
The horizontal distance from her eyes to the ruler is 60 cm.

Calculate an estimate of the height of the tree.

$33.\dot{3} \times 30 = 1000cm$

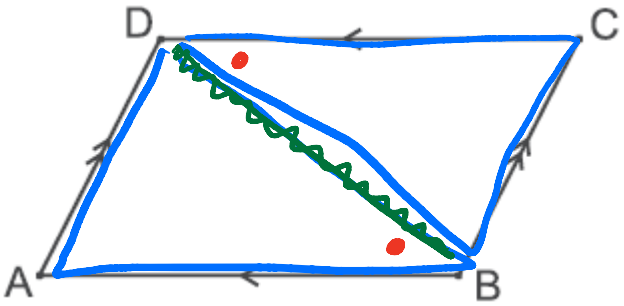
$\div 100 = 10m$

(a) ..... 10 ..... m [3]

(b) Give two reasons why this method may not be suitable to estimate the height of a very tall building.

- likely to be inaccurate due to large scale factors and distances
- would have to stand a long way from the building [2]

6. *ABCD* is a parallelogram.



Prove that triangle *ABD* is congruent to triangle *CDB*.

DB = shared side

CDB = DBA because alternate angles are equal

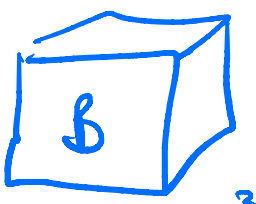
DC = AB ∴ ABD is congruent to CDB (SAS)

[3]

7. Solid **A** and solid **B** are mathematically similar.
The ratio of the surface area of solid A to the surface area of solid B is 4:9

The volume of solid B is 405cm^3.

Show that the volume of solid A is 120cm^3.



405cm^3

surface area

A	B
4	9
1	$\frac{9}{4}$

(÷4 ÷4)

area SF = $\frac{9}{4}$ so length SF = $\frac{3}{2}$

$405 \div \left(\frac{3}{2}\right)^3 = 120cm^3$

[3]

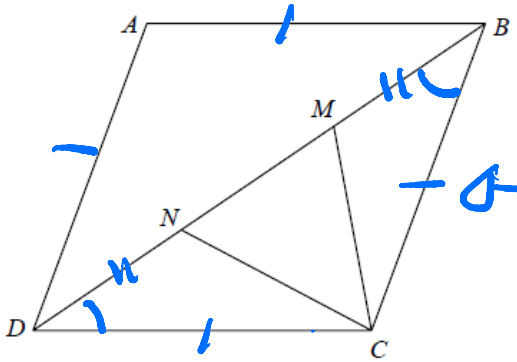
8. Two spheres have radii in the ratio 5 : 3

Circle the ratio of their volumes.

5 : 3 15 : 9 25 : 9 125 : 27

[1]

9. *ABCD* is a rhombus.



M and *N* are points on *BD* such that *DN* = *MB*.

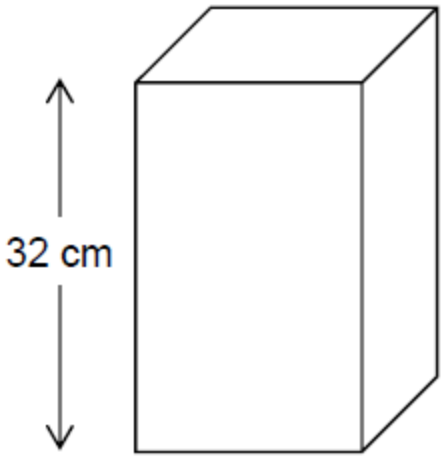
Prove that triangle *DNC* is congruent to triangle *BMC*.

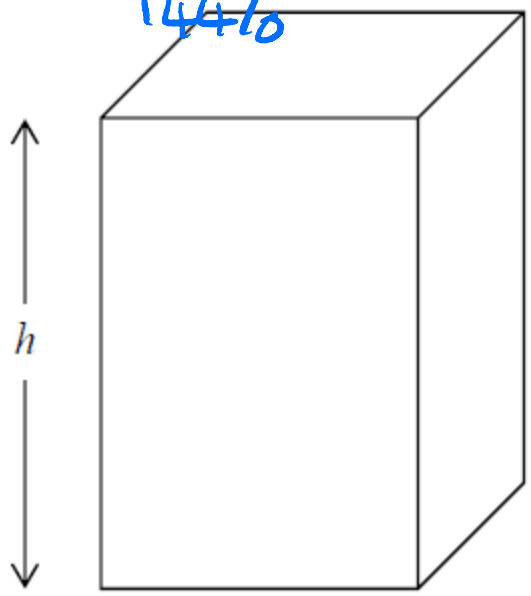
[3]

10. Two boxes are made with card.

The boxes are similar cuboids.

The smaller box has height 32 cm





It takes 44% more card to make the larger box.

Work out the height, *h*, of the larger box.

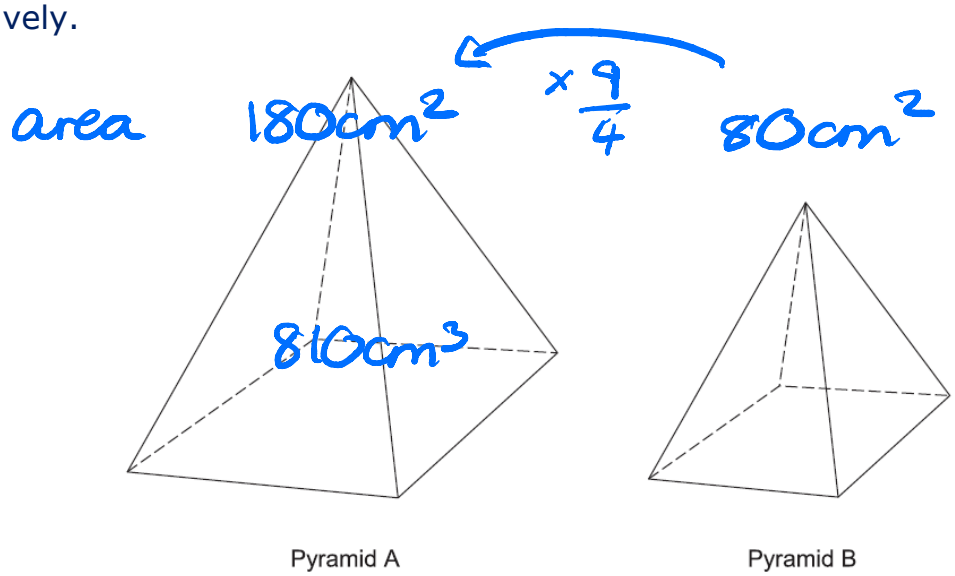
area SF = 1·44

length SF = $\sqrt{1·44}$ = 1·2

32 × 1·2 = 38·4cm

[4]

11. Two similar pyramids A and B have surface areas 180 cm^2 and 80 cm^2 respectively.



Pyramid A Pyramid B

The volume of pyramid A is 810 cm^3.

Show that the volume of pyramid B is 240 cm^3.

area SF = $\frac{9}{4}$

length SF = $\sqrt{\frac{9}{4}}$ = $\frac{3}{2}$

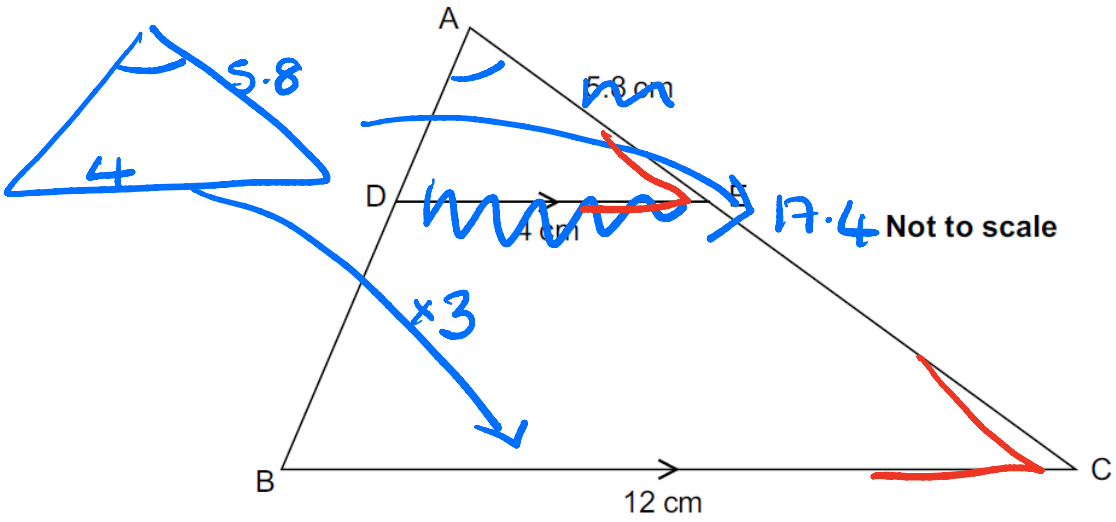
Volume SF = $\left(\frac{3}{2}\right)^3$ = $\frac{27}{8}$ = 3·375

810 ÷ 3·375 = 240cm^3

QED

[5]

12. In the diagram BC is parallel to DE.



a) Prove that triangle ABC is similar to triangle ADE.

AED = ACB corresponding angles are equal
ADE = ABC corresponding angles are equal
DAC = shared angle.
so all angles are preserved

[3]

b) Calculate the length of AC.

5.8 × 3

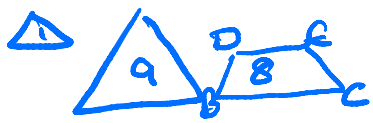
b) ..... 17.4 ..... cm

[2]

c) Find the ratio

area of quadrilateral DBCE : area of triangle ABC.

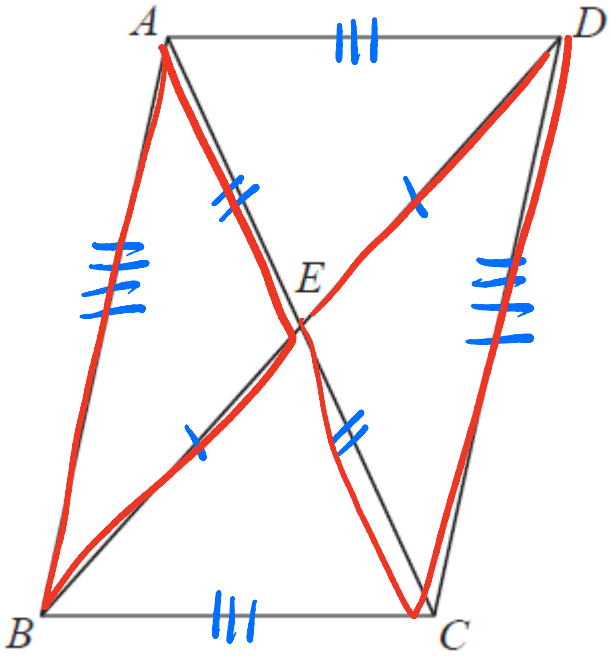
length SF = 3
so area SF = $3^2 = 9$
area ADE : area ABC
1 : 9



area DBCE area ABC

c) 8 ..... : 9 ..... [3]

13 *ABCD* is a parallelogram.



E is the point where the diagonals *AC* and *BD* meet.

Prove that triangle *ABE* is congruent to triangle *CDE*.

AE = CE diagonals bisect
AEB = DEC vertically opposite angles are equal
ED = EB diagonals bisect ∴ they are congruent SAS [3]

14 Mark has made a clay model.
He will now make a clay statue that is mathematically similar to the clay model.

The model has a base area of 6cm^2
The statue will have a base area of 253.5cm^2

Mark used 2kg of clay to make the model.

Clay is sold in 10kg bags.
Mark has to buy all the clay he needs to make the statue.

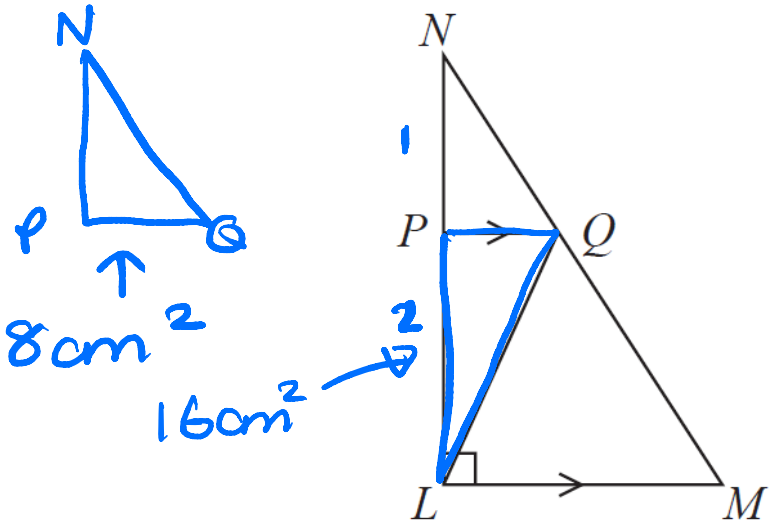
2kg
6cm^2 253.5cm^2

How many bags of clay will Mark need to buy?

area SF = 253.5 ÷ 6 = 42.25
length SF = $\sqrt{42.25}$ = 6.5
Vol SF = 6.5^3

2×6.5^3 = 549.25kg 549.25 ÷ 10 = 54.925 bags [3]
55 bags

15.



LMN is a right-angled triangle.

Angle NLM = 90°

PQ is parallel to LM.

The area of triangle PNQ is 8 cm²

The area of triangle LPQ is 16 cm²

Work out the area of triangle LQM.

16 ÷ 8 = 2

they share a base so

PL = 2NP

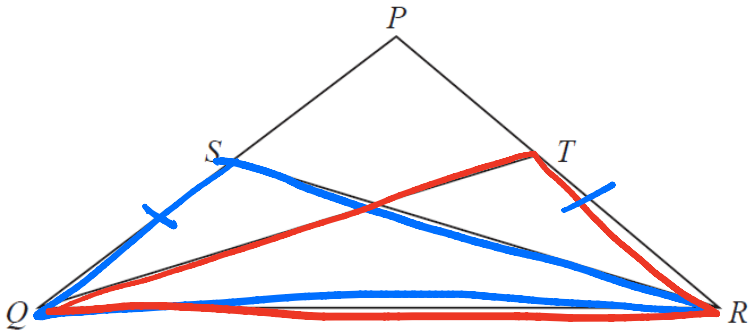
area LMN = 8 × 3² = 72cm²

LPQ = 72 − (8 + 16) = 72 − 24

48

..... cm² [4]

16.



$PQ = PR$.

S is the midpoint of PQ.

T is the midpoint of PR.

Prove triangle QTR is congruent to triangle RSQ.

PRQ = PQR base angles on an isosceles

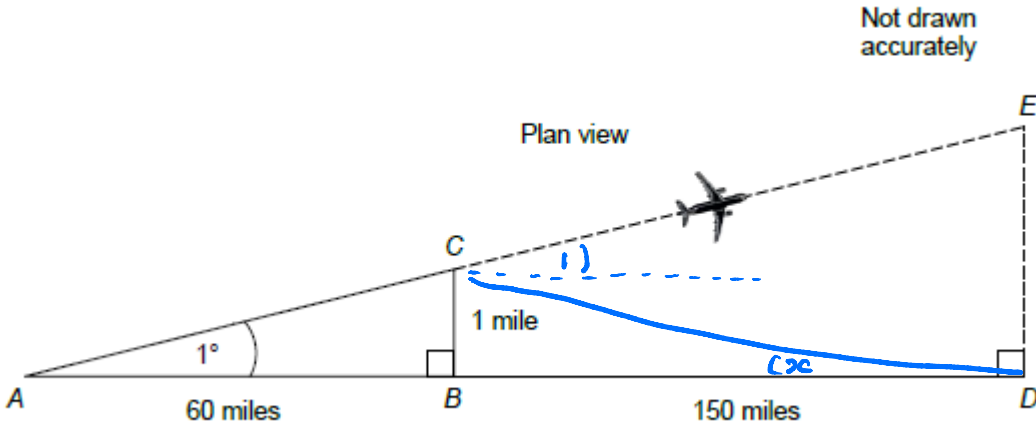
QR = shared side.

QS = TR sides are bisected ∴ they are congruent SAS

[3]

17. The pilot of an aircraft wants to fly from A to D.

The aircraft flies from A to E, 1° off course.



$\tan x = \frac{1}{150}$

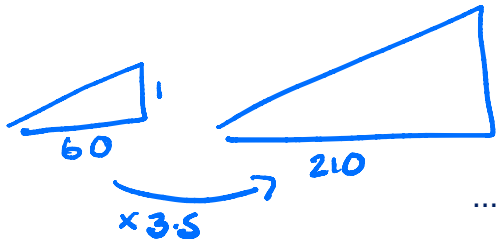
$x = 0.38°...$

$+ 1$

$= 1.38°...$

a) The distance BC is 1 mile.

Work out the distance DE.



3.5 ..... miles [2]

b) How should the aircraft have turned at C to fly directly towards D?

Tick a box.

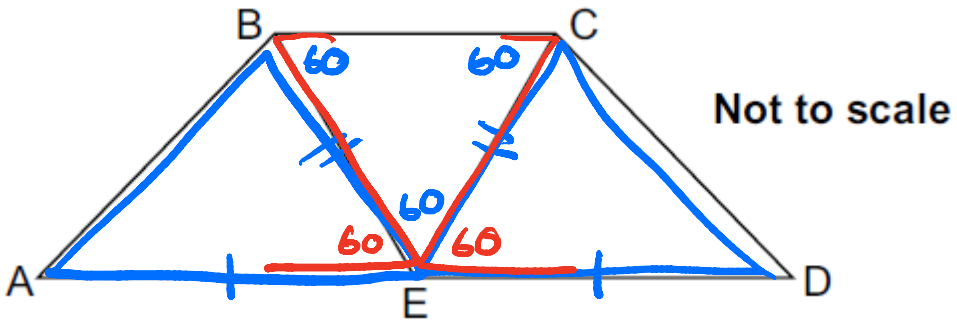
1° clockwise 	☒
between 1° and 2° clockwise	☑
2° clockwise	☐
more than 2° clockwise	☐

[1]

18. The diagram shows trapezium ABCD.

E is the midpoint of AD.

BCE is an equilateral triangle.

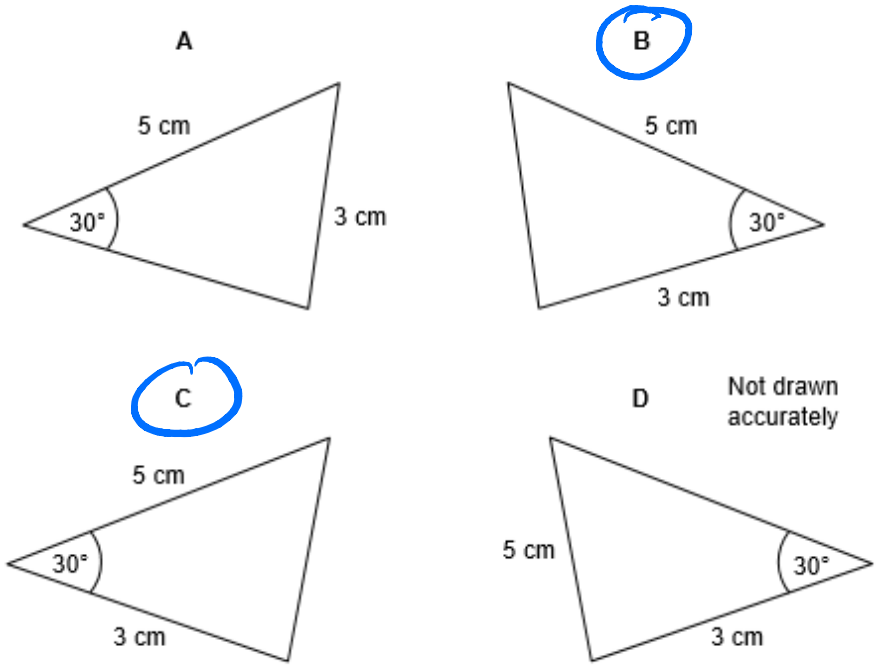


Prove that triangle ABE is congruent to triangle DCE.

BE = EC sides of an equilateral triangle.
BEA = CED = 60° alternate angles are equal.
AE = ED E is the midpoint of AD ∴ they are congruent SAS.

[4]

19. Here are four triangles.



a) Which two triangles are congruent? Circle your answers.

A B C D

[1]

b) Circle the reason for your answer to part (a).

SSS ASA SAS RHS

[1]

CREDITS AND NOTES

Q	Awarding Body	Q	Awarding Body	Q	Awarding Body
1	WJEC Eduqas	8	AQA	15	Pearson Edexcel
2	AQA	9	Pearson Edexcel	16	Pearson Edexcel
3	AQA	10	AQA	17	AQA
4	Pearson Edexcel	11	OCR	18	OCR
5	OCR	12	OCR	19	AQA
6	OCR	13	Pearson Edexcel		
7	Pearson Edexcel	14	Pearson Edexcel		

Notes:

These questions have been retyped from the original sample/specimen assessment materials and whilst every effort has been made to ensure there are no errors, any that do appear are mine and not the exam board s (similarly any errors I have corrected from the originals are also my corrections and not theirs!).

Please also note that the layout in terms of fonts, answer lines and space given to each question does not reflect the actual papers to save space.

These questions have been collated by me as the basis for a GCSE working party set up by the GLOW maths hub - if you want to get involved please get in touch. The objective is to provide support to fellow teachers and to give you a flavour of how different topics "could" be examined. They should not be used to form a decision as to which board to use. There is no guarantee that a topic will or won't appear in the "live" papers from a specific exam board or that examination of a topic will be as shown in these questions.

Links:

AQA http://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-8300

OCR http://ocr.org.uk/gcsemaths

Pearson Edexcel http://qualifications.pearson.com/en/qualifications/edexcel-gcses/mathematics-2015.html

WJEC Eduqas http://www.eduqas.co.uk/qualifications/mathematics/gcse/

Contents:

This version contains questions from:

AQA – Sample Assessment Material, Practice set 1 and Practice set 2

OCR – Sample Assessment Material and Practice set 1

Pearson Edexcel – Sample Assessment Material, Specimen set 1 and Specimen set 2

WJEC Eduqas – Sample Assessment Material