Forming & Solving Equations (H)

JustMaths

Name:	*Lisa Woods*
Total Marks:	

1. The diagram below shows a composite shape formed by joining two rectangles.
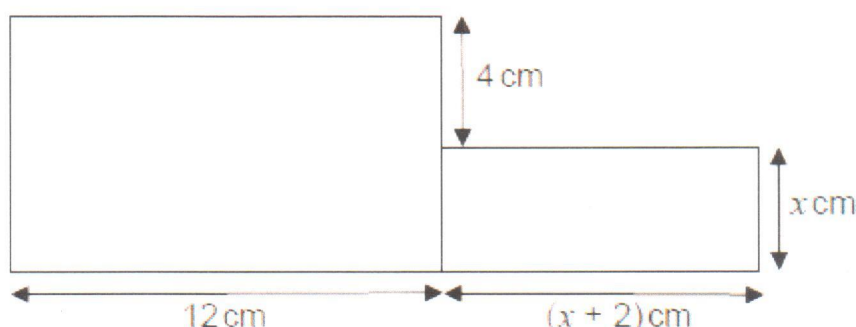
$$\frac{12(4+x) = x(x+2)}{4}$$

$3(4+x) = x(x+2)$

$12 + 3x = x^2 + 2x$

$x^2 - x - 12 = 0$ *x cannot be negative*

$(x + 3)(x - 4)$ $\therefore x = 4$

$x = 4$

$x = -3$

4 cm

x cm

12 cm

(*x* + 2) cm

Diagram not drawn to scale

The area of the larger rectangle is 4 times the area of the smaller rectangle.

Calculate the dimensions of the smaller rectangle.

You must justify any decisions that you make.

Dimensions are 4cm x 6cm

check 12 x 8   4x6

= 96   24

[7]

2. The diagram shows a square.

All the lengths are measured in centimetres.

Diagram not drawn to scale

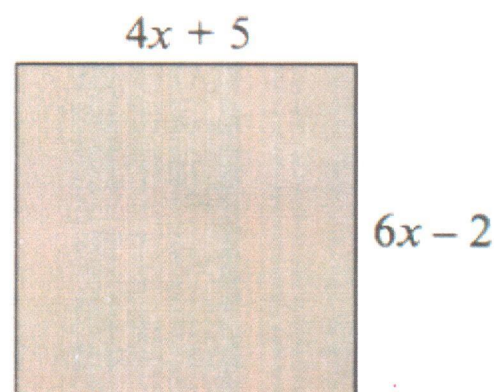Use an algebraic method to find the length of one side of the square.

$4x + 5$

$6x - 2$

Diagram not drawn to scale

$4x + 5 = 6x - 2$

$(-4x) \qquad (-4x)$

$5 = 2x - 2$

$(+2) \qquad (+2)$

$2x = 7$

$x = 3.5$

$\therefore 4x + 5$

$4 \times 3.5 + 5$

$= 19cm$

[5]

3. (a) Solve $\frac{3}{x} = 12$

$$\frac{3}{x} = 12$$
$$12x = 3$$
$$x = \frac{3}{12} \qquad \therefore x = \frac{1}{4}$$

[1]

(b) Solve $9x - 4 = 7(x + 2)$.

$$9x - 4 = 7x + 14$$
$$(-7x) \qquad (-7x)$$
$$2x - 4 = 14$$
$$(+4) \quad (+4)$$
$$2x = 18$$
$$x = 9$$

[3]

4. ABCH is a square.

HCFG is a rectangle.

CDEF is a square.

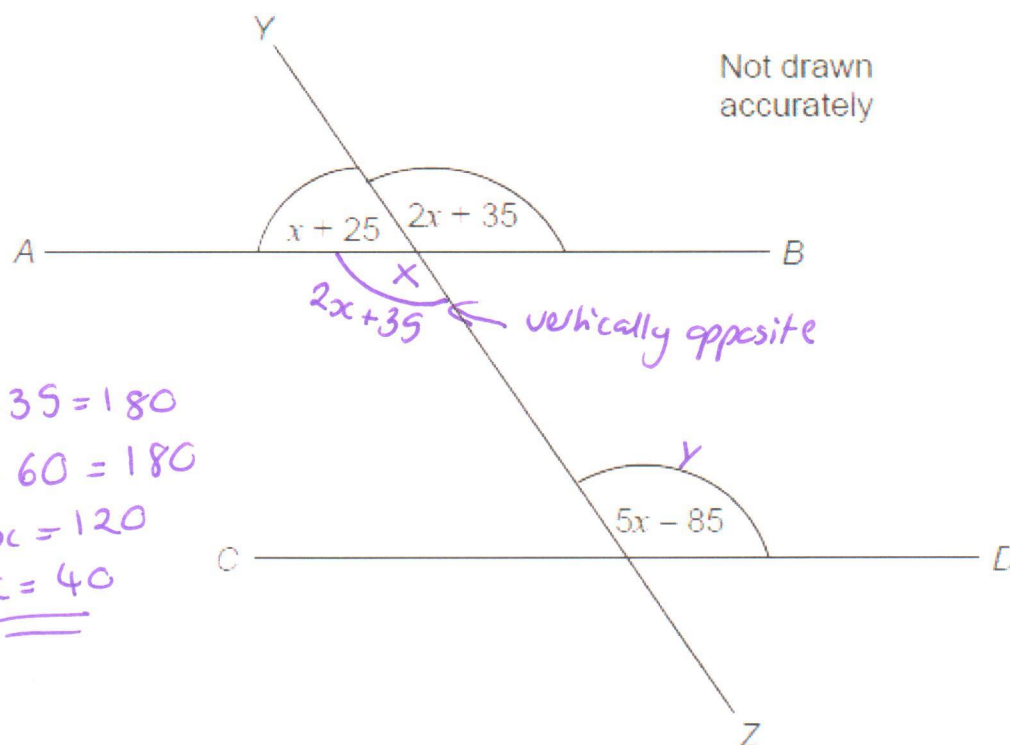Not drawn accurately

$(x - 3)$ cm

$x + 3$

3

3 cm
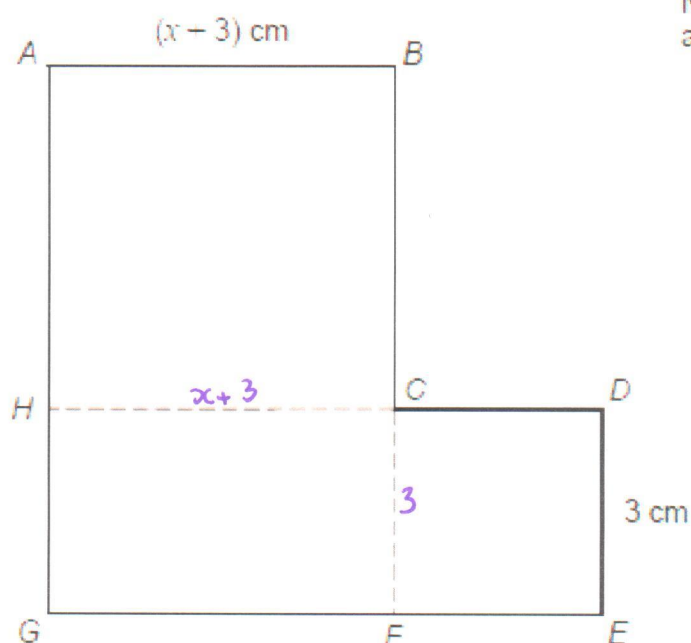
They are joined to make an L-shape.

Show that the total area of the L-shape, in cm², is $x^2 + 9x + 27$

$$(x+3)^2 + 3(x+3) + 3^2$$
$$x^2 + 6x + 9 + 3x + 9 + 9$$
$$= x^2 + 9x + 27$$

[4]

5. AB, CD and YZ are straight lines.

All angles are in degrees.

Y

Not drawn
accurately

$x + 25$ | $2x + 35$

A ——————————————————— B

$2x + 35$ x ← *vertically opposite*

$x + 25 + 2x + 35 = 180$
$3x + 60 = 180$
$3x = 120$
$x = 40$

Y

$5x - 85$

C ——————————————————— D

Z

Show that AB is parallel to CD. *If parallel then X = Y by alternate angles are equal*

$2x + 35$ $5x - 85$ *∴ lines are parallel*
$2 \times 40 + 35 = 115°$ $5 \times 40 - 85$
                  $= 115°$

[4]

6. Here is a parallelogram.
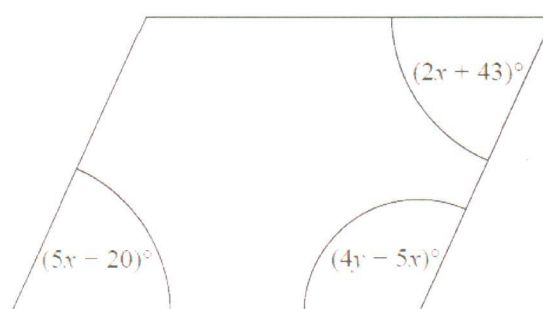
$(2x + 43)°$

$(5x - 20)°$ $(4y - 5x)°$

Work out the value of x and the value of y.

$5x - 20 = 2x + 43$ $∴$ $4y - 5x = \dfrac{190}{2}$
$3x - 20 = 43$ $4y - 5 \times 21 = 95$
$3x = 63$ $4y - 105 = 95$
$x = 21$ $4y = 200$ $∴ y = 50$

$2x + 43$ $360 - 85 - 85$ x = ...21...
$2 \times 21 + 43 = 85$ $= 190$ y = ...50...

[5]

7. Kieran, Jermaine and Chris play football.

- Kieran has scored 8 more goals than Chris.
- Jermaine has scored 5 more goals than Kieran.
- Altogether they have scored 72 goals.

How many goals did they each score?

$K = C + 8$ ∴ $C = K - 8$

$J = K + 5$ ∴

$K + C + J = 72$

$K + (K-8) + (K+5) = 72$

$K + K - 8 + K + 5 = 72$

$3K - 3 = 72$ $3K = 75$

$K = 25$

Kieran ....25..........

Jermaine ....30..........

Chris ....17..........

[5]

8. ABCD is a rectangle.

EFGH is a trapezium.

$2(3x+4) + 2(4x) = x - 3 + 5x + 5x + 7x - 3$

$6x + 8 + 8x = 18x - 6$

$14x + 8 = 18x - 6$

$8 = 4x - 6$
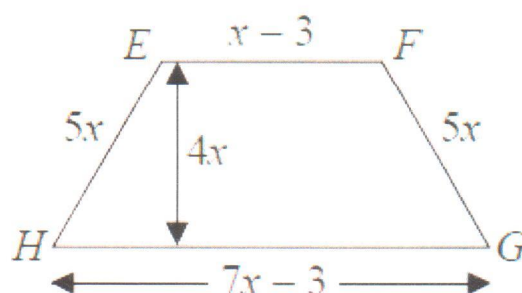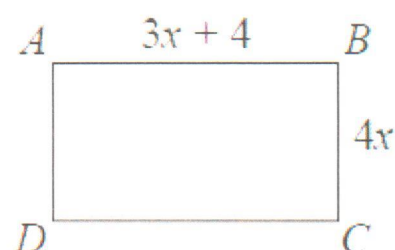
$14 = 4x$

$\underline{\underline{x = 3.5}}$

All measurements are in centimetres.

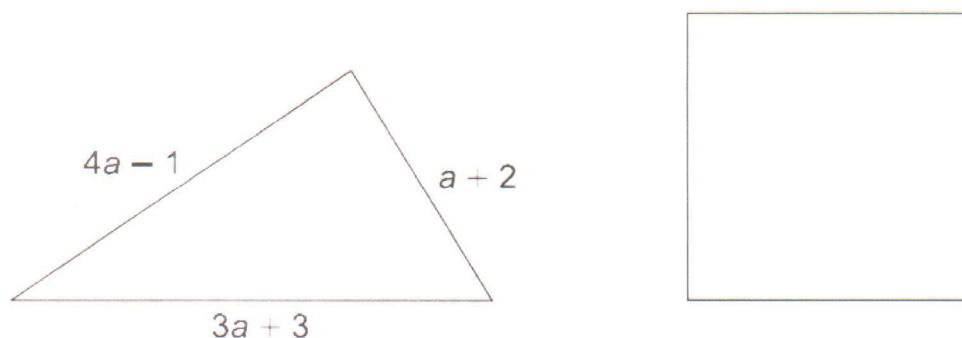The perimeters of these two shapes are the same.

Work out the area of the rectangle.

$[(3 \times 3.5) + 4] \times 4 \times 3.5$

$= 203$

..........203.......... .cm² [5]

9. The perimeter of the triangle is the same length as the perimeter of the square.

4a − 1   a + 2

3a + 3

Find an expression for the length of one side of the square in terms of a.

Give your answer in its simplest form.

$4a - 1 + a + 2 + 3a + 3$

$8a + 4$

$\dfrac{8a + 4}{4} = 2a + 1$

.........$2a + 1$......... [4]

10.   Alexander, Reiner and Wim each watch a different film.

• Alexander's film is thirty minutes longer than Wim's film.

• Reiner's film is twice as long as Wim's film.

• Altogether the films last 390 minutes.

How long is each of their films?

$A = 30 + W$

$R = 2W$
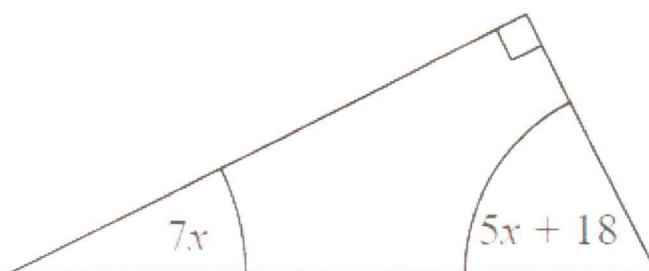
$A + R + W = 390$

$30 + W + 2W + W = 390$

$4W = 360$

$\underline{\underline{W = 90}}$

$R = 2 \times 90$

$= 180$

$A = 90 + 30$

$= 120$

Alexander's film ....120........ minutes

Reiner's film ....180........ minutes

Wim's film ....90........ minutes [4]

11. The diagram shows a right-angled triangle.

All the angles are in degrees.

Work out the size of the smallest angle of the triangle.

$7x + 5x + 18 = 90$
$12x + 18 = 90$
$12x = 72$
$x = 6$

$7 \times 6 = 42$
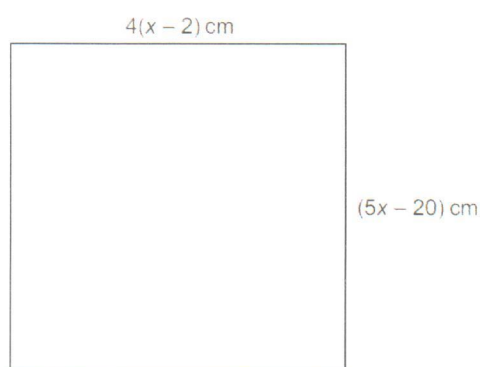$5 \times 6 + 18 = 48$

$5x + 18$ smallest
$\underline{48}$

[3]

12. This is a square.

4(x − 2) cm   **Not to scale**

(5x − 20) cm

Work out the length of the side of the square.

$4(x-2) = 5x - 20$
$4x - 8 = 5x - 20$
$-8 = x - 20$
$12 = x$

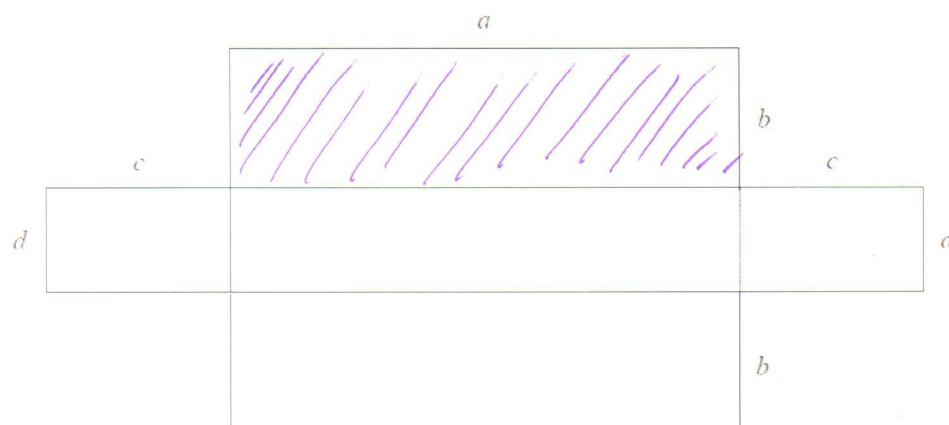$5x - 20$
$5 \times 12 - 20 = 40$

40

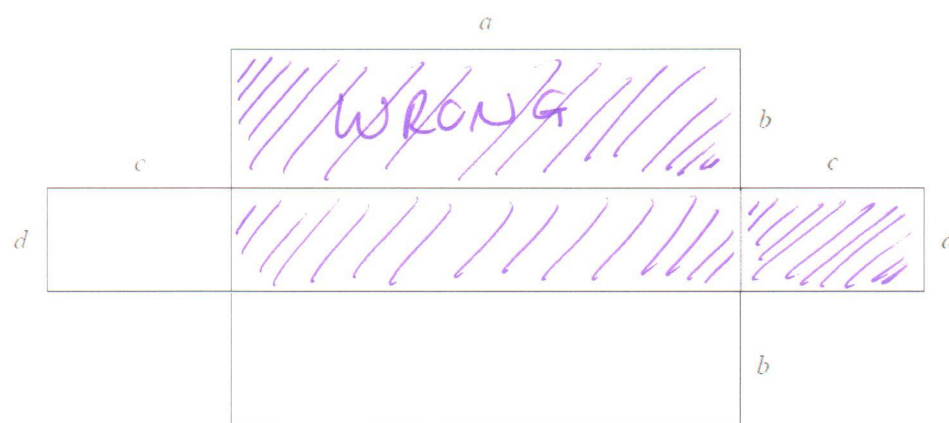.. cm [5]

13. A shape is made from rectangles.

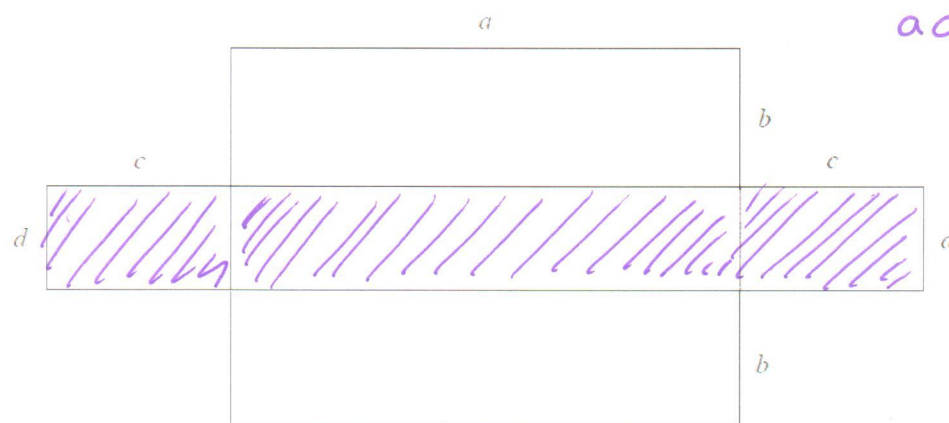(a) On the diagram below shade an area represented by the expression ab

a

c *b* *c*

d *d*

b

[1]

(b) On the diagram below shade an area represented by the expression ad + cd

a

c WRONG *b* *c*

d *d*

b

[1]

(c) On the diagram below shade the area represented by the expression d(a + 2c)
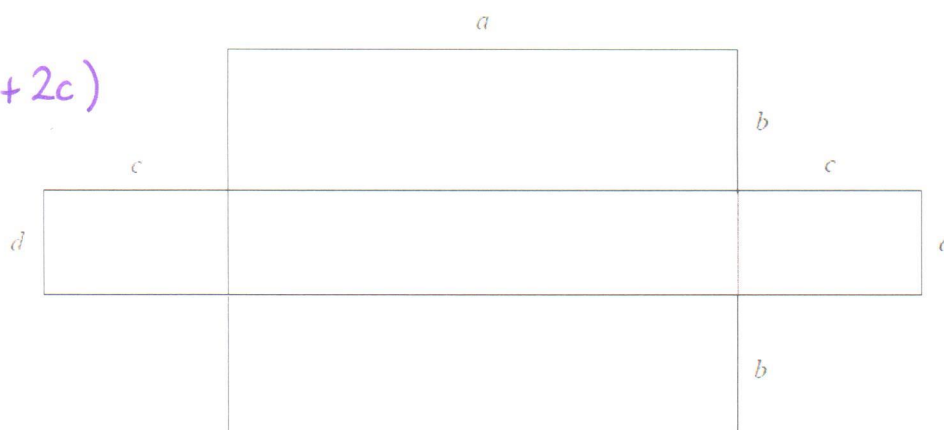
ad + 2cd

a

c *b* *c*

d *d*

b

[1]

(d) Write down an expression for the area of the whole shape.

$2ab + d(a + 2c)$

[1]

14 The diagram shows a square.

$(7x – 3)$ cm
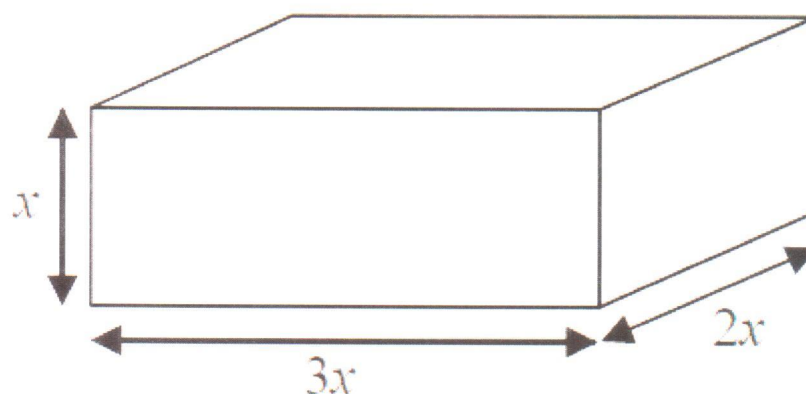
$7x - 3 = 3x + 3$
$4x = 6$
$x = 1.5$
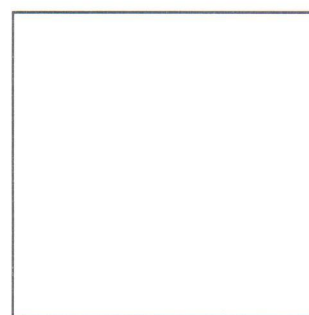
$3(x + 1)$ cm

Work out the length of one side of the square.

$3 \times 2.5 = 7.5$ cm

[4]

15 Here is a cuboid.

All measurements are in centimetres.

x is an integer.

The total volume of the cuboid is less than 900 cm³
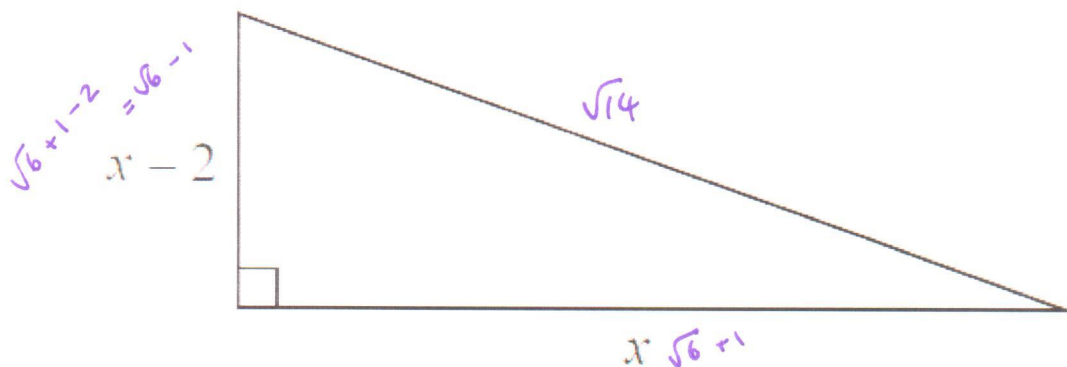
Show that $x \leq 5$

$x \times 3x \times 2x < 900$

$6x^3 < 900$

$x^3 < 150$

$x < \sqrt[3]{150}$

$x < 5.31329$ ∴ as x is an integer   [3]

it must be ≤ 5.

16. Here is a right-angled triangle.

$\sqrt{6+1-2} = \sqrt{6}-1$

$\sqrt{6}+1-2 = \sqrt{6}-1$

$x - 2$

$\sqrt{14}$

$x\sqrt{6}+1$

All measurements are in centimetres.

The area of the triangle is 2.5 cm².

Find the perimeter of the triangle.

Give your answer correct to 3 significant figures.

You must show all of your working.

$\dfrac{x(x-2)}{2} = 2.5$

$x(x-2) = 5$

$x^2 - 2x - 5 = 0$

$(x-1)^2 - 6 = 0$

$x - 1 = \pm\sqrt{6}$

$x = \pm\sqrt{6} + 1$

must be $+\sqrt{6}$ since x can't be negative

$x = \sqrt{6} + 1$

missing side $\sqrt{(\sqrt{6}+1)^2 + (\sqrt{6}+1-2)^2}$

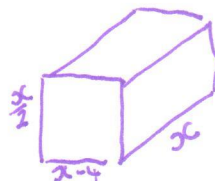$= \sqrt{14}$

$\sqrt{14} + \sqrt{6} + 1 + \sqrt{6} - 1$

8.64

.... cm [6]

17. A cuboid has length x cm.

The width of the cuboid is 4 cm less than its length.

The height of the cuboid is half of its length.

a) The surface area of the cuboid is 90 cm².

Show that $2x^2 - 6x - 45 = 0$.

$2\left(\dfrac{x}{2} \times (x-4)\right) + 2\left(\dfrac{x}{2} \times x\right) + 2\left(x \times (x-4)\right) = 90$

$x^2 - 4x + x^2 + 2x^2 - 8x = 90$

$\underset{x^2-4x}{}$

$4x^2 - 12x = 90$

$4x^2 - 12x - 90 = 0 \quad (\div 2)$

$2x^2 - 6x - 45 = 0$

[5]

find x first
$2x^2 - 6x - 45 = 0$

$x = \dfrac{6 \pm \sqrt{36 + 360}}{4}$

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$x = 6.475$ $x = -3.475$
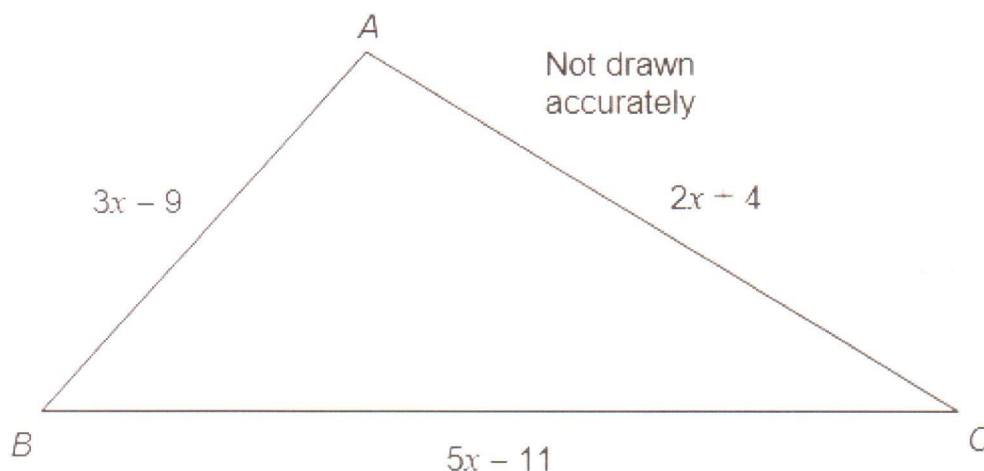(can't be negative)

b) Work out the volume of the cuboid.

$\therefore$ Volume $= x \times \dfrac{x}{2} \times (x - 4)$

$= \dfrac{x^3}{2} - 2x^2$

$= \dfrac{6.475^3}{2} - 2 \times (6.475)^2 = 51.88$

(b) .... 51.9 .... cm³ [6]

18. In this question all lengths are in centimetres.

Not drawn
accurately

$3x - 9$ $2x + 4$

$5x - 11$

Given $AB : BC = 1 : 2$

show that $AC : BC = 3 : 4$

[5]

$\dfrac{2}{3}(3x - 9) = \dfrac{1}{3}(5x - 11)$

$6x - 18 = 5x - 11$

$x - 18 = -11$

$x = 7$

$AC = 2x + 4$
$= \underline{\underline{18}}$

$BC = 5x - 11$
$= \underline{\underline{24}}$

$AC : BC$
$18 : 24$
$\underline{\underline{3 : 4}}$