GCSE Subject Level Conditions and Requirements for Mathematics

April 2016

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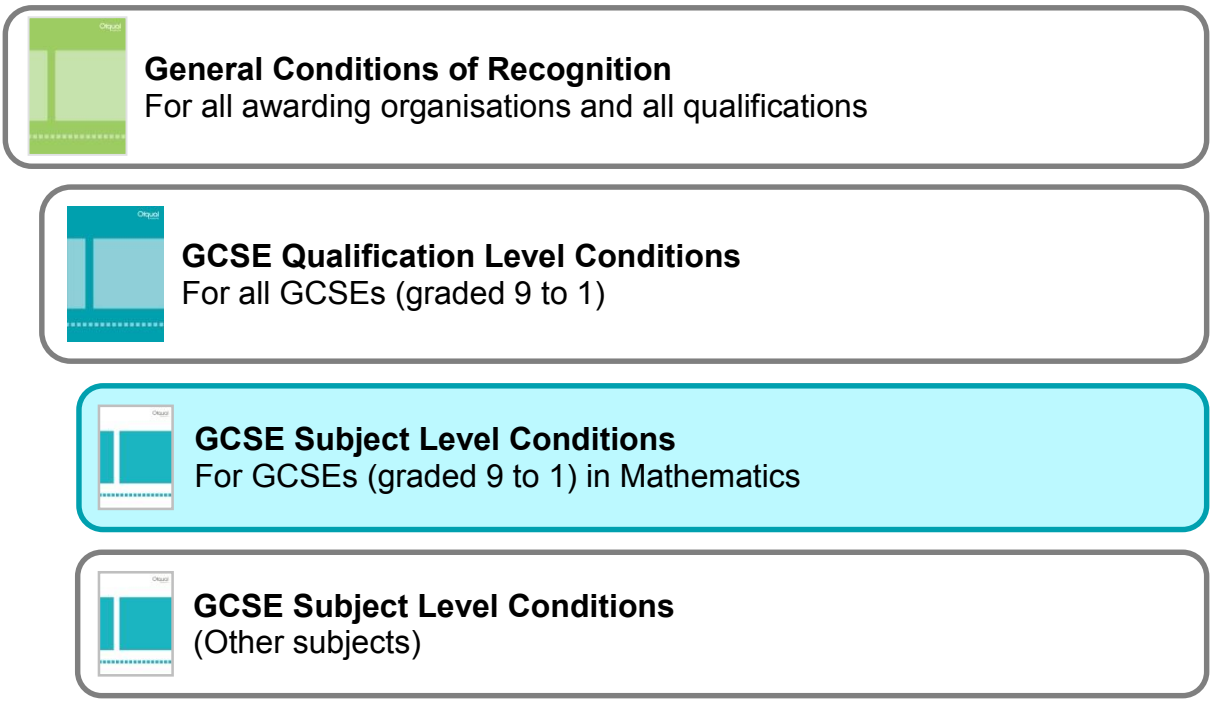
Appendix 1 – Revisions to this document

Introduction

About this document

This document (highlighted in the figure below) is part of a suite of documents which sets out the regulatory requirements for awarding organisations offering GCSE qualifications (graded from 9 to 1).



We have developed these requirements with the intention that GCSE qualifications (graded from 9 to 1) should fulfil the following purposes:

- To provide evidence of students' achievements against demanding and fulfilling content;
- To provide a strong foundation for further academic and vocational study and for employment; and
- To provide (if required) a basis for schools and colleges to be held accountable for the performance of all of their students.

Requirements set out in this document

This document sets out the GCSE Subject Level Conditions for Mathematics. It also sets out our requirements in relation to:

- interpretation of subject content – awarding organisations must comply with these requirements under Condition GCSE(Mathematics)1.1(c); and

- tiering of assessment – awarding organisations must comply with these requirements under Condition GCSE(Mathematics)3.1.

Appendix 1 reproduces the requirements in relation to subject content and assessment objectives for GCSE Mathematics[1], as published by the Department for Education. Awarding organisations must comply with these requirements under Condition GCSE(Mathematics)1.1.

With respect to GCSE qualifications (graded from 9 to 1) in Mathematics, awarding organisations must also comply with:

- our *General Conditions of Recognition*,[2] which apply to all awarding organisations and qualifications; and
- our *GCSE Qualification Level Conditions*;[3] and
- all relevant Regulatory Documents.[4]

With respect to GCSE qualifications graded from A* to G, awarding organisations must continue to comply with the General Conditions of Recognition, and the relevant Regulatory Documents.

Revisions to this document

We have revised this document since it was originally published (see Appendix 1 for details), most recently in April 2016.

The April 2016 version of this document replaces all previous versions of *GCSE Subject Level Conditions and Requirements for Mathematics* with effect from 10.01am on Tuesday 12 April 2016. The following Conditions and requirements published in previous versions of this document are withdrawn, and will cease to have effect, from that time:

- Condition GCSE(Mathematics)4.2
- Condition GCSE(Mathematics)4.3
- Use of calculators – GCSE Qualifications (graded 9 to 1) in Mathematics

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[1] www.gov.uk/government/publications/gcse-mathematics-subject-content-and-assessment-objectives

[2] www.gov.uk/government/publications/general-conditions-of-recognition

[3] www.gov.uk/government/publications/gcse-9-to-1-qualification-level-conditions

[4] www.gov.uk/government/publications/regulatory-documents-list

Summary of requirements

Subject Level Conditions	
GCSE(Mathematics)1	Compliance with content requirements
GCSE(Mathematics)2	Timing of assessments
GCSE(Mathematics)3	Assessing the full range of abilities
GCSE(Mathematics)4	Use of Calculators

Interpretation of subject content
Requirements in relation to subject content for GCSE Qualifications in Mathematics

Tiering requirements
Requirements for tiering of assessments in GCSE Mathematics

Appendix 1 – Subject content and assessment objectives (published by Department for Education)
GCSE Mathematics: subject content and assessment objectives

Subject Level Conditions

GCSE Subject Level Conditions for Mathematics

Condition GCSE(Mathematics)1 Compliance with content requirements

GCSE(Mathematics)1.1 In respect of each GCSE Qualification in Mathematics which it makes available, or proposes to make available, an awarding organisation must –

(a) comply with the requirements relating to that qualification set out in the document published by the Secretary of State entitled 'GCSE Mathematics: subject content and assessment objectives'[5], document reference DFE-00233-2013,

(b) have regard to any recommendations or guidelines relating to that qualification set out in that document, and

(c) interpret that document in accordance with any requirements, and having regard to any guidance, which may be published by Ofqual and revised from time to time.

[5] www.gov.uk/government/publications/gcse-mathematics-subject-content-and-assessment-objectives

Condition GCSE(Mathematics)2 Timing of assessments

GCSE(Mathematics)2.1 Condition GCSE6.1 does not apply to any GCSE Mathematics qualification which an awarding organisation makes available or proposes to make available.

GCSE(Mathematics)2.2 An awarding organisation must ensure that in respect of each GCSE Mathematics qualification which it makes available each Learner completes the Assessments by Examination to be taken for that qualification during the months of –

(a) May and/or June, or

(b) November,

in any single year.

GCSE(Mathematics)2.3 In respect of an assessment for a GCSE Mathematics qualification which it makes available an awarding organisation must take all reasonable steps to ensure that each Learner who takes an Assessment by Examination during the month of November has reached at least the age of 16 on or before 31 August in the same calendar year as the assessment

Condition GCSE(Mathematics)3 Assessing the full range of abilities

GCSE(Mathematics)3.1 In respect of a GCSE Mathematics qualification that an awarding organisation makes available, or proposes to make available –

(a) Condition GCSE1.1 does not apply, and

(b) the awarding organisation must ensure that the qualification, and each assessment for it, complies with any requirements which may be published by Ofqual and revised from time to time

Condition GCSE(Mathematics)4 Use of Calculators

GCSE(Mathematics)4.1 In designing and setting the assessments for a GCSE Mathematics qualification which it makes available, or proposes to make available, an awarding organisation must ensure that between 33 and 50 per cent of the total marks available in those assessments are allocated to questions or tasks which must be completed by Learners without the use of a Calculator.

Interpretation of subject content

Requirements in relation to subject content for GCSE Qualifications in Mathematics

The subject content and assessment objectives for GCSE Qualifications (graded 9 to 1) in Mathematics is set out in the Department for Education's *Mathematics GCSE subject content and assessment objectives*, document reference DFE-00233-2013 (the 'Content Document').[6]

Condition GCSE(Mathematics)1.1(c) requires awarding organisations to interpret the Content Document in line with any requirements published by Ofqual.

We set out our requirements for the purposes of Condition GCSE(Mathematics)1.1(c) below.

Weighting of assessment objectives

The Content Document specifies percentage weightings for each of the three assessment objectives AO1 to AO3 for both higher and foundation tier assessments.

In respect of each GCSE Qualification in Mathematics which it makes available, or proposes to make available, an awarding organisation must interpret the assessment objective weightings specified in the Content Document as requiring it to ensure that –

(a) in each assessment series, the weighting allocated to each assessment objective at each of the foundation and higher tiers differs by no more than ±3 per cent from that specified in the Content Document,

(b) taking together those assessments over the first two years of awarding (i.e. the first four assessment series) the weightings specified in the Content Document are achieved for each of the foundation and higher tiers, and

(c) taking together those assessments over each subsequent and discrete two-year period the weightings specified in the Content Document are achieved for each of the foundation and higher tiers.

For the purposes of paragraph (c) above, the reference to each subsequent and discrete two-year period means years 3-4 of awarding, years 5-6 of awarding, and so on. As each two-year period will be discrete there will not be any overlap between periods. Thus, an awarding organisation is not required to achieve the weightings

[6] www.gov.uk/government/publications/gcse-mathematics-subject-content-and-assessment-objectives

specified in the Content Document, in years 2-3 or years 4-5 of awarding, for example.

Tiering requirements

Tiering requirements – GCSE Qualifications (graded 9 to 1) in Mathematics

Condition GCSE(Mathematics)3.1(b) allows us to specify requirements relating to assessing the full range of abilities for GCSE Qualifications (graded 9 to 1) in Mathematics.

In accordance with Condition GCSE(Mathematics)3.1(b) an awarding organisation must design, deliver and award a GCSE Qualification in Mathematics that it makes available, or proposes to make available, in accordance with the requirements set out below.

Use of the overlapping tiers model

1. GCSE Qualifications in mathematics must be tiered. An awarding organisation must design and set the assessments for a GCSE Qualification in Mathematics that it makes available, or proposes to make available, using an overlapping tiers model.

2. Such a model must use two tiers – a foundation tier and a higher tier – and each assessment must be designed and set in such a way as to fall within one of those two tiers only.

Preclusion of mixed tier entry

3. An awarding organisation must ensure that each Learner is permitted to take assessments in either the foundation tier or the higher tier only.

Grades available at each tier

4. The questions or tasks in foundation tier assessments must be targeted at the Level of Demand required for the award of GCSE grades 1 to 5. A Learner who takes foundation tier assessments must be awarded a grade within the range of 1 to 5, or be unclassified.

5. The questions or tasks in higher tier assessments must be targeted at the Level of Demand required for the award of grades 4 to 9. A Learner who takes higher tier assessments must be awarded a grade within the range of 4 to 9, or be unclassified. However, if the mark achieved by such a Learner is a small number of marks below the 4/3 grade boundary, that Learner may be awarded a grade 3.

6. The 3/U grade boundary for higher tier assessments is provisionally set by subtracting half the mark interval between the 5/4 and 4/3 grade boundaries (rounding up half marks) from the 4/3 boundary.

The targeting of grades within each assessment

7. It is important that, taken together, the assessments across both tiers, and also within each tier, adequately cover all levels of ability and permit sufficient discrimination between Learners' attainment.

8. Also, the proportions in which grades are targeted within each tier must promote accurate classification across different Learners' attainment within that tier.

9. An awarding organisation must, therefore, take all reasonable steps to ensure that the marks available for each assessment within the foundation tier are targeted as follows.

    - 50% of those marks must be targeted at a Level of Demand consistent with grade 1 to the lower part/half of grade 3.

    - 50% of marks must be targeted at a Level of Demand consistent with the upper part/half of grade 3 to grade 5.

10. An awarding organisation must also take all reasonable steps to ensure that the marks available for each assessment within the higher tier are targeted as follows.

    - 50% of those marks must be targeted at a Level of Demand consistent with grades 4 to 6.

    - 50% of marks must be targeted at a Level of Demand consistent with grades 7 to 9.

11. The assessments for each individual tier must be designed so as to enable sound grading judgements about Learners' attainment and so as to promote public confidence in these qualifications. In designing assessments, an awarding organisation must take all reasonable steps to ensure, at each tier, that Learners achieving the lowest targeted grade have demonstrated attainment with regard to a sufficient range of the subject requirements, in terms of the areas of content, content domains and the assessment objectives. Equally, an awarding organisation must take all reasonable steps to ensure, at each tier, that Learners achieving the higher targeted grades must have demonstrated attainment with regard to suitably stretching and challenging requirements, in terms of the areas of content, content domains and the assessment objectives.

The overlap at grades 4 and 5

12. Grades 4 and 5 are the only grades that are designed to be accessible by Learners who have taken either foundation tier or higher tier assessments.

(Although Learners taking higher tier assessments may be awarded a grade 3, as indicated at paragraph 5 above, this grade is not actively targeted in these assessments.)

13. To help make sure that the level of attainment indicated by grades 4 and 5 is consistent, regardless of the tier for which a Learner is entered, an awarding organisation must ensure that at least 20% of the marks available in the assessments for each tier are made available through questions that are common to both tiers. These questions must be targeted at a Level of Demand consistent with grades 4 and 5.

14. To avoid a breach of confidentiality, an awarding organisation must ensure that the foundation and higher tier assessments that use common questions are designed to be taken simultaneously by all relevant Learners.

15. An awarding organisation must ensure that the approach it takes to tiering is transparent. Each awarding organisation must demonstrate in its assessment strategy how, between the two tiers, it will secure comparability between the levels of attainment required for grades 4 and 5. The securing of such comparability should include, but is not restricted to:

    - the use of marks from common questions;
    - the proportion of marks for each tier that are targeted at the overlapping grades; and
    - the level of attainment that Learners achieving these grades at the different tiers must demonstrate in relation to the areas of content, content domains and assessment objectives.

The Secretary of State's content requirements

16. The document published by the Secretary of State entitled *GCSE Mathematics: Subject Content and Assessment Objectives*, document reference DFE-00233-2013 (the 'Content Document'), outlines three areas of content which must be covered in all GCSE Qualifications in Mathematics.

    Area 1: Content with regard to which it is intended that all Learners taking the qualification should be confident and competent by the end of their GCSE course. This content is shown in standard font in the Content Document and must be assessed in both higher and foundation tier assessments.

    Area 2: Content with which all students taking the qualification are intended to be at least familiar by the end of their GCSE course. This content is underlined in the Content Document and must be assessed in both higher and foundation tier assessments. Students taking higher tier assessments should be expected

to be confident and competent with this content by the end of their GCSE course, and those assessments must reflect that expectation.

Area 3: Content with which only the most highly attaining Learners will be expected to be confident and competent by the end of their GCSE course. This content is shown in **bold font** in the Content Document and must be assessed in higher tier assessments only.

17. An awarding organisation must demonstrate in its assessment strategy, how its approach to sampling will ensure that:

    - the foundation tier will assess all of the content in Areas 1 and 2; and
    - the higher tier will assess all of the content in Areas 1, 2 and 3,

    in as few successive Assessment Series[7] as it is possible to achieve this and in such a way that the content to be targeted in a given Assessment Series is not unduly predictable.

18. For each tier, these Areas do not need to be targeted in equal proportions in each Assessment Series, either by individual assessments or by the assessments for an Assessment Series taken together. In addition, they do not need to be targeted in the same proportions in every Assessment Series.

19. In addition, an awarding organisation must demonstrate in its assessment strategy how it will design its assessments to cover the content in each Area and how it will ensure that assessments are not predictable.

20. Questions and tasks in different tiers which cover the same Area may vary, in particular in regard to the Level of Demand and the assessment objectives targeted.

The weighting of content domains

21. The Content Document sets out content domains which must be covered in all specifications for GCSE Qualifications in Mathematics.

22. An awarding organisation will not be able to cover all aspects of each content domain in the assessments for a given tier in any single Assessment Series. It

[7] For the purposes of these requirements, an 'Assessment Series' means the assessments to be taken by a particular Learner for a GCSE Qualification in Mathematics. For clarity, the assessments taken by Learners may vary, depending on the tier for which they have been entered and any possible routes through the qualification. These assessments will be taken by each Learner during one of the periods outlined in Condition GCSE(Mathematics)2.2.

will instead design assessments that sample from the content domains. When it does so, it must not produce assessments that are predictable and must demonstrate in its assessment strategy how this is to be achieved.

23. The assessments for a GCSE Qualification in Mathematics should achieve the balance of coverage of the content domains set out in the table below. The weightings reflect the size and demand of the content domains and take into account the ability range for each tier.

Domain area	Weighting of marks per Assessment Series	
	Foundation tier	Higher tier
Number	25%	15%
Algebra	20%	30%
Ratio, proportion and rates of change	25%	20%
Geometry	15%	20%
Probability and statistics	15%	15%

24. An awarding organisation should apply these weightings, subject to a +/-3% tolerance for each domain area, across the assessments for each tier in each Assessment Series. An awarding organisation that does not achieve the weighting set out in the table across the assessments in a given Assessment Series should achieve those weightings in as few Assessment Series as is possible.

25. An awarding organisation should keep its approach to the application of any tolerance under review to identify any unintended consequences.

Subject content and assessment objectives (published by Department for Education)

Department for Education

Mathematics

GCSE subject content and assessment objectives

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Introduction

GCSE subject criteria set out the knowledge, understanding, skills and assessment objectives common to all GCSE specifications in a given subject. They provide the framework within which awarding organisations create the detail of their specifications, so ensuring progression from key stage 3 national curriculum requirements and the possibilities for development into advanced-level study.

Subject aims and learning outcomes

This document sets out the learning outcomes and content coverage required for GCSE specifications in mathematics. In subjects such as mathematics, where topics are taught in progressively greater depth over the course of key stage 3 and key stage 4, GCSE outcomes may reflect or build upon subject content which is typically taught at key stage 3. There is no expectation that teaching of such content should be repeated during the GCSE course where it has already been taught effectively at an earlier stage.

GCSE specifications in mathematics should provide a broad, coherent, satisfying and worthwhile course of study. They should encourage students to develop confidence in, and a positive attitude towards mathematics and to recognise the importance of mathematics in their own lives and to society. They should also provide a strong mathematical foundation for students who go on to study mathematics at a higher level post-16.

GCSE specifications in mathematics should enable students to:

1. develop fluent knowledge, skills and understanding of mathematical methods and concepts
2. acquire, select and apply mathematical techniques to solve problems
3. reason mathematically, make deductions and inferences and draw conclusions
4. comprehend, interpret and communicate mathematical information in a variety of forms appropriate to the information and context.

Students should be aware that mathematics can be used to develop models of real situations and that these models may be more or less effective depending on how the situation has been simplified and the assumptions that have been made. Students should also be able to recall, select and apply mathematical formulae (see Appendix).

Subject content

GCSE specifications in mathematics should reflect the aims and learning outcomes outlined above, and should include the knowledge, understanding and skills listed below, giving due consideration to the assessment objectives. The essential subject content outlined here provides the framework for developing a coherent study at GCSE.

This content sets out the full range of content for GCSE specifications in mathematics. Awarding organisations may, however, use any flexibility to increase depth, breadth or context within the specified topics or to consolidate teaching of the subject content.

Students can be said to have confidence and competence with mathematical content when they can apply it flexibly to solve problems.

The expectation is that:

- All students will develop confidence and competence with the content identified by standard type
- All students will be assessed on the content identified by the standard and the underlined type; more highly attaining students will develop confidence and competence with all of this content
- Only the more highly attaining students will be assessed on the content identified by **bold** type. The highest attaining students will develop confidence and competence with the **bold** content.

The distinction between standard, underlined and **bold** type applies to the content statements only, not to the assessment objectives or to the mathematical formulae in the appendix.

Scope of study

GCSE specifications in mathematics should require students to:

Number

Structure and calculation

1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)

3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
4. use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem
5. apply systematic listing strategies **including use of the product rule for counting**
6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5***;* estimate powers and roots of any given positive number**
7. calculate with roots, and with integer **and fractional** indices
8. calculate exactly with fractions, **surds** and multiples of π**; simplify surd expressions involving squares (e.g.** $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$**) and rationalise denominators**
9. calculate with and interpret standard form A x 10^n, where $1 \leq A < 10$ and n is an integer.

Fractions, decimals and percentages

10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 or $\frac{3}{8}$)**; change recurring decimals into their corresponding fractions and vice versa**
11. identify and work with fractions in ratio problems
12. interpret fractions and percentages as operators.

Measures and accuracy

13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
16. apply and interpret limits of accuracy**, including upper and lower bounds**

Algebra

Notation, vocabulary and manipulation

1. use and interpret algebraic notation, including:
    - ab in place of $a \times b$
    - $3y$ in place of $y + y + y$ and $3 \times y$
    - a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$
    - $\frac{a}{b}$ in place of $a \div b$
    - coefficients written as fractions rather than as decimals
    - brackets
2. substitute numerical values into formulae and expressions, including scientific formulae
3. understand and use the concepts and vocabulary of expressions, equations, formulae, identities inequalities, terms and factors
4. simplify and manipulate algebraic expressions (including those involving surds **and algebraic fractions**) by:
    - collecting like terms
    - multiplying a single term over a bracket
    - taking out common factors
    - expanding products of two **or more** binomials
    - factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares**; factorising quadratic expressions of the form** $\boldsymbol{ax^2 + bx + c}$
    - simplifying expressions involving sums, products and powers, including the laws of indices
5. understand and use standard mathematical formulae; rearrange formulae to change the subject
6. know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**
7. where appropriate, interpret simple expressions as functions with inputs and outputs**; interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'.**

Graphs

8. work with coordinates in all four quadrants

9. plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel **and perpendicular lines**; find the equation of the line through two given points, or through one point with a given gradient
10. identify and interpret gradients and intercepts of linear functions graphically and algebraically
11. identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically **and turning points by completing the square**
12. recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$**, exponential functions** $y = k^x$ **for positive values of** k**, and the trigonometric functions (with arguments in degrees)** $y = \sin x$, $y = \cos x$ **and** $y = \tan x$ **for angles of any size**
13. **sketch translations and reflections of a given function**
14. plot and interpret graphs (including reciprocal graphs **and exponential graphs**) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
15. **calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts**
16. **recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point.**

Solving equations and inequalities

17. solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
18. solve quadratic equations (**including those that require rearrangement**) algebraically by factorising**, by completing the square and by using the quadratic formula**; find approximate solutions using a graph
19. solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph
20. **find approximate solutions to equations numerically using iteration**
21. translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.
22. solve linear inequalities in one **or two** variable**(s), and quadratic inequalities in one variable**; represent the solution set on a number line**, using set notation and on a graph**

Sequences

23. generate terms of a sequence from either a term-to-term or a position-to-term rule
24. recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 **or a surd**) **and other sequences**
25. deduce expressions to calculate the n^{th} term of linear **and quadratic** sequences.

Ratio, proportion and rates of change

1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
2. use scale factors, scale diagrams and maps
3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
4. use ratio notation, including reduction to simplest form
5. divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
6. express a multiplicative relationship between two quantities as a ratio or a fraction
7. understand and use proportion as equality of ratios
8. relate ratios to fractions and to linear functions
9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
10. solve problems involving direct and inverse proportion, including graphical and algebraic representations
11. use compound units such as speed, rates of pay, unit pricing, density and pressure
12. compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

13. understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; **construct and** interpret equations that describe direct and inverse proportion
14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
15. **interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts**
16. set up, solve and interpret the answers in growth and decay problems, including compound interest **and work with general iterative processes.**

Geometry and measures

Properties and constructions

1. use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
2. use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
3. apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
4. derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
5. use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
6. apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
7. identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional **and negative** scale factors)

8. **describe the changes and invariance achieved by combinations of rotations, reflections and translations**
9. identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
10. **apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results**
11. solve geometrical problems on coordinate axes
12. identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
13. construct and interpret plans and elevations of 3D shapes.

Mensuration and calculation

14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
15. measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
16. know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
17. know the formulae: circumference of a circle $= 2\pi r = \pi d$, area of a circle $= \pi r^2$; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids
18. calculate arc lengths, angles and areas of sectors of circles
19. apply the concepts of congruence and similarity, including the relationships between lengths, **areas and volumes** in similar figures
20. know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin\theta = \frac{opposite}{hypotenuse}$, $\cos\theta = \frac{adjacent}{hypotenuse}$ and $\tan\theta = \frac{opposite}{adjacent}$; apply them to find angles and lengths in right-angled triangles **and, where possible, general triangles** in two **and three** dimensional figures
21. know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0^0, 30^0, 45^0, 60^0$ and 90^0; know the exact value of $\tan\theta$ for $\theta = 0^0, 30^0, 45^0$ and 60^0
22. **know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$, to find unknown lengths and angles**
23. **know and apply $\text{Area} = \frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle.**

Vectors

24. describe translations as 2D vectors
25. apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **use vectors to construct geometric arguments and proofs**

Probability

1. record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
2. apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
3. relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale
4. apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
5. understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
6. enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams
7. construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
8. calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
9. **calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams.**

Statistics

1. infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
2. interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use
3. **construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use**

4. interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
    - appropriate graphical representation involving discrete, continuous and grouped data, **including box plots**
    - appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers**, quartiles and inter-quartile range**)
5. apply statistics to describe a population
6. use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

Assessment objectives

Assessment Objectives		Weighting	
		Higher	Foundation
AO1	**Use and apply standard techniques** Students should be able to: • accurately recall facts, terminology and definitions • use and interpret notation correctly • accurately carry out routine procedures or set tasks requiring multi-step solutions	40%	50%
AO2	**Reason, interpret and communicate mathematically** Students should be able to: • make deductions, inferences and draw conclusions from mathematical information • construct chains of reasoning to achieve a given result • interpret and communicate information accurately • present arguments and proofs • assess the validity of an argument and critically evaluate a given way of presenting information Where problems require candidates to 'use and apply standard techniques' or to independently 'solve problems' a proportion of those marks should be attributed to the corresponding Assessment Objective	30%	25%
AO3	**Solve problems within mathematics and in other contexts** Students should be able to: • translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes • make and use connections between different parts of mathematics • interpret results in the context of the given problem • evaluate methods used and results obtained • evaluate solutions to identify how they may have been affected by assumptions made Where problems require candidates to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding Assessment Objective.	30%	25%

Appendix: Mathematical formulae

1. Formulae included in the subject content. Candidates are expected to know these formulae; they must not be given in the assessment.

The quadratic formula

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Circumference and area of a circle

Where r is the radius and d is the diameter:

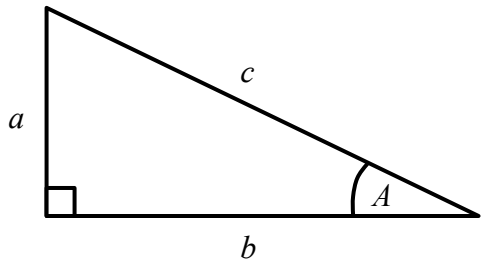
$$\text{Circumference of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$

Pythagoras's theorem

In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$



Trigonometry formulae

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

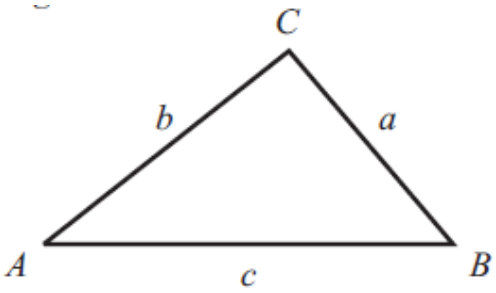
$$\sin A = \frac{a}{c}, \quad \cos A = \frac{b}{c}, \quad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides

$$\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{cosine rule: } a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area} = \frac{1}{2}ab\sin C$$



2. <u>The following formulae are not specified in the content but should be derived or informally understood by candidates. These formulae must not be given in the examination.</u>

Perimeter, area, surface area and volume formulae

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a+b)h$$

$$\text{Volume of a prism} = \text{area of cross section} \times \text{length}$$

Compound interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

Where $P(A)$ is the probability of outcome A and $P(B)$ is the probability of outcome B:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B)P(B)$$

3. Formulae that candidates should be able to use, but need not memorise. These can be given in the exam, either in the relevant question, or in a list from which candidates select and apply as appropriate.

Perimeter, area, surface area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

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Appendix 1 – Revisions to this document

The table below sets out all revisions to this document since it was first published, and the dates on which those revisions came into force.

Revision	Date in force
■ Condition GCSE(Mathematics)4 – Use of Calculators (amended) ■ Use of calculators – GCSE Qualifications (graded 9 to 1) in Mathematics (withdrawn)	12 April 2016
Requirements in relation to subject content (new)	30 June 2015
Tiering requirements (new)	23 May 2014
First published	9 April 2014

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